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Incentives for quality improvement efforts coordination in supply chains with partial cost allocation contract

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In this paper, we consider quality improvement efforts coordination in a two-stage decentralised supply chain with a partial cost allocation contract. The supply chain consists of one supplier and one manufacturer, both of which produce defective products. Two kinds of failure cost occur within the supply chain: internal and external. The supplier and the manufacturer determine their individual quality levels to maximise their own profits. We propose a partial cost allocation contract, under which the external failure cost is allocated between the manufacturer and the supplier at different rates based on information derived from failure root cause analysis. If the quality levels of the supplier and the manufacturer are observable, we show that the partial cost allocation contract coordinates the supply chain, provided that the failure root cause analysis does not erroneously identify the manufacturer's fault as the supplier's, and the supplier does not take responsibility for the manufacture's fault. In the single moral hazard model, where only the quality level of the supplier is unobservable, the optimal share rates require the supplier to take some responsibility for the manufacture's fault. However, in the double moral hazard model, where quality levels of the supplier and the manufacturer are unobservable to each other, the optimal share rates require the supplier not to take responsibility for the manufacturer's fault. It is noted that the root cause analysis conducted by the manufacturer may have its disadvantage in attributing the fault to the supplier when both sides are at fault. We also propose a contract based on the dual root cause analysis to reduce the supplier's penalty cost. Numerical results illustrate that the partial cost allocation contract satisfies the fairness criterion compared with the traditional cost allocation contract.

Keywords: supply chain management; quality management; supply chain coordination; moral hazard; partial cost allocation contract

1. Introduction

Recently, product quality crises have captured the attention of consumers and society alike. For example, the 2008 tainted milk powder crisis in China caused thousands of children to suffer from kidney illness, and the Sanlu Group, an established dairy company in China, went bankrupt because of its involvement in this quality scandal. This crisis has subsequently eroded customers' confidence in Chinese-made milk powder products. Canon, the world's largest camera manufacturer, announced in 2009 quality problems with its flagship EOS 5D Mark II model camera, and the company has not yet resolved the problems. Months later, Nikon, another well-known camera manufacturer, admitted quality problems in its Nikon D5000 model camera. Evidently, quality problems affect customer satisfaction and tarnish firm reputation, and it takes a long-time and substantial resources to restore corporate reputation and customer confidence.

It is generally known that quality has a great impact on a manufacturer's reputation and customer confidence; firms have been taking actions to improve their quality management systems. As manufacturers increasingly outsource their components to external suppliers, coordination of quality improvement efforts among members of the supply chain has become an important issue to assure the quality of the final products. The 2011 Chinese Automobile and After-sale Service Quality Report[1] documented that 22.23% of customer complaints referred to quality issues, while 48.9% referred to the after-sale service related to quality issues. According to the same report, among quality complaints lodged, 24.88% concerned the engine, 24.30% the body accessories, 12.75% the transmission system and the remaining 38.07% other components. The findings of the report illustrate that the quality of outsourced components has a significant impact on the quality of the final product, which in turn affects customer satisfaction with the final product.

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Therefore, in the context of manufacturing outsourcing, enhancing the coordination of quality improvement efforts among members of supply chains becomes a challenge to the manufacturers. To address this issue, there is an expanding body of literature considering quality control issues from the supply chain's perspective, see, e.g. Zhu, Zhang, and Tsung (2007) and Chao, Iravani, and Savaskan (2009). Chao, Iravani, and Savaskan (2009) first introduce the partial allocation contract to allocate the external failure cost case by case between the supplier and the manufacturer based on the information provided by a failure root cause analysis. Before the introduction of the partial allocation contract, fixed share rates were commonly adopted, whereby the external failure cost is allocated between the manufacturer and the supplier at fixed share rates. It is therefore interesting to compare the performance of a partial allocation contract against that of a fixed share rates contract.

In this paper, we consider the coordination of quality improvement efforts between a manufacturer and a supplier in a two-stage supply chain. Each player produces defective products with a probability determined by his/her own quality level. We consider two kinds of failure cost, namely, internal failure cost and external failure cost, see Baiman, Fischer, and Rajan (2001), and Balachandran and Radhakrishnan (2005). The internal failure cost is the penalty for delivering a defective component from the supplier to the manufacturer, while the external failure cost is the penalty for selling a defective product from the manufacturer to the customer. The manufacturer conducts inspections to identify the cause for external failures, which is called failure root cause analysis, see Chao, Iravani, and Savaskan (2009). Many manufacturers have adopted failure root cause analysis in their operations. For instance, IBM has designated a department to find the root cause once an external failure is reported by customer. Furthermore, IBM reserves the right to ask for compensation from the supplier if the supplier is at fault. When the final product is separable or modularised, like computers, televisions, cell phones and automobiles, it is common to resort to failure root cause analysis to figure out who should take responsibility for external failures. Without failure root cause analysis, the failure cost is normally shared between the manufacturer and supplier at fixed share rates. We propose a partial cost allocation contract which specifies different share rates of the external cost based on the information from the failure root cause analysis. The partial cost allocation contract is a consequence-based contract which allocates the failure cost based on who takes responsibility for the failure. We illustrate that the partial cost allocation contract is able to coordinate the supply chain to reach a profit that can be achieved in a centralised supply chain under some conditions, while the fixed share rates contact is unlikely to achieve the same profit level. Therefore, the partial cost allocation contract, which subsumes the fixed share rates contract as a special case, is indeed an improvement over the fixed share rates contract. Considering that the root cause analysis conducted by the manufacturer attributes the fault to the supplier when both sides are at fault, we also propose another contract based on the dual root cause analysis to reduce the penalty of the supplier.

The remainder of the paper is organised as follows: In Section 2, related literature is reviewed. Section 3 presents the basic model and the corresponding notation and terminologies. Section 4 focuses on the centralised system. The moral hazard models are presented in Section 5. We further propose another partial cost allocation contract with dual root cause analyses in Section 6. Lastly, we conclude the paper and suggest future research directions in Section 7.

2. Literature review

A large body of literature has studied quality issues from an operations management perspective, see Porteus (1986), Rosenblatt and Lee (1986), and Yano and Lee (1995). Recently, there is a growing body of literature analysing quality issues from a supply chain coordination perspective. This literature can be broadly classified into two categories, namely, quality competition games, e.g. Federgruen and Yang (2009), and quality coordination games, e.g. Chao, Iravani, and Savaskan (2009).

Existing literature mostly studies quality management within a single firm; however, quality issues may arise in the supply chain. Yang et al. (2012) propose a dual-sourcing option to decrease supply risk when suppliers face the possibility of disruption. An optimal procurement contract provides the manufacturer with the opportunity of choosing between diversification and competition advantage. Babich and Tang (2012) develop some mechanisms to deter suppliers from adulteration, including the deferred payment mechanism, inspection and combined mechanisms. Existing work closely related to our work is Baiman, Fischer, and Rajan (2000), who study a double moral hazard model in a supply chain consisting of one supplier and one buyer with incomplete information. The supplier's quality level and the buyer's inspection quality are both unobservable. They show that there exist incentivised contracts to induce the centralised optimal choice, (i.e. the choice maximising the total profit of the supply chain in different settings). Subsequently, in similar settings, Lim (2001) considers a game in the adverse selection setting, where the supplier type characterised by different quality levels is not observable to the buyer, and the buyer offers a contract menu that specifies the inspection scheme and warranty scheme to induce the supplier to report its true quality type. Lim finds that a supplier with a quality level higher than a certain critical level prefers a warranty scheme with no inspection, while a supplier with a quality level lower than the critical level prefers an inspection scheme. Differing from Lim (2001), our paper applies hazard models and proposes a partial cost allocation contract to induce the supplier to choose a quality level which maximises the whole supply chain profit. Zhu, Zhang, and Tsung (2007) consider a Stackelberg game in which both the buyer and the supplier have an incentive to invest in quality improvement efforts. The buyer facing a deterministic demand first decides its order batch and quality improvement efforts, and then the supplier decides its own production lot size and improvement efforts in the second stage. The costs include both inventory-related and quality improvement costs. They investigate the roles of the buyer and the supplier in the quality improvement efforts. Recently, Lee, Rhee, and Cheng (2013) study supply chain coordination with quality uncertainty; they assume that demand is sensitive to the product quality. Other related studies are Reyniers and Tapiero (1995), and Gurnani and Gerchak (2007). The aforementioned papers consider quality issues associated with the supplier and do not take quality issues associated with the manufacturer into consideration. Our paper differs from prior research in that we consider a system where both supplier and manufacturer produce defective products. We aim to propose a mechanism to improve the quality level in the entire supply chain.

In a work that considers a two-stage supply chain consisting of a supplier and a manufacturer, Baiman, Fischer, and Rajan (2001) propose a signalling game in which the supplier has private information about its own quality, which is either of a high type or a low type, and the manufacturer determines its own inspection quality and processing quality, given its belief on the supplier's quality type. The authors study various signalling games where external failure, or internal failure or both are contractible. Wu et al. (2011) analyse quality information sharing in a system consisting of one buyer and two suppliers with quality uncertainty; they verify that information sharing benefits the buyer and the suppliers. Shi et al. (2014) consider a system with one manufacturer and two suppliers. One is a strategic supplier with long lead time and high quality, while the other is an urgent supplier with short lead time and low quality. The quality level of the urgent supplier is private information. The researchers show that the suppliers are better off from information sharing, but the manufacturer is worse off. Related studies are Balachandran and Radhakrishnan (2005), and Chao, Iravani, and Savaskan (2009). The aforementioned works consider a case where both supplier and buyer produce defective components. Balachandran and Radhakrishnan (2005) develop a single moral hazard model and a double moral hazard model with either the supplier's or the manufacturer's quality being unobservable. The manufacturer acting as the principal proposes a penalty/warranty contract to induce the supplier to choose the supply chain profit-maximisation quality level. The contract specifies that the supplier is penalised when it takes responsibility for the failure based on information gathered from incoming inspection or internal failure. They find that a contract based on information from incoming inspection coordinates the supply chain to achieve the centralised optimal quality level. For the fairness criterion, on the other hand, a contract based on information from external failure performs better. Similarly to Balachandran and Radhakrishnan (2005), our paper considers both single and double moral hazard models. Our paper differs from Balachandran and Radhakrishnan (2005) because they assume that the manufacturer takes responsibility for its own fault and shares part of the external failure cost caused by the supplier; we assume that the external failure cost is shared between the supplier and buyer based on information derived from failure root cause analysis. In our proposed contract, the share rates vary from case to case based on who is responsible for external failure. We illustrate that the partial cost allocation contract is much more flexible than the contract studied in Balachandran and Radhakrishnan (2005), especially when the wholesale price is exogenously determined and cannot be negotiated. Chao, Iravani, and Savaskan (2009) introduce two new recall cost sharing contracts, namely, the selective root cause analysis contract and the partial cost allocation contract. They show that the two contracts can coordinate the supply chain to reach the centralised optimal profit level in a batch order and a batch recall setting. In contrast, we focus on finding the external cost allocation scheme in a case-by-case setting based on failure root cause analysis. We show that the partial cost allocation contract can coordinate the supply chain to reach the centralised optimal profit level with both complete and incomplete information.

3. Model description

Consider a supply chain consisting of a risk-neutral supplier and a risk-neutral manufacturer. The supplier produces components and delivers them to the manufacturer for further processing. The manufacturer then assembles the components and sells them to the customers. The production processes of both the supplier and the manufacturer are not perfect. Both of them will produce defective products with different probabilities. We assume that the market demand is exogenous and constant, representing a situation where the sales of products are relatively stable, and can be seen as constant demand. Without loss of generality, we assume demand is normalised to one. The supplier makes quality improvement efforts to reach its desired quality level β, which is defined as the probability that a delivered component is without defect. Then the probability that the component is detective is $1 - \beta$. It costs the supplier $S(\beta)$ per unit component to achieve the desired quality level β. Similar to Baiman, Fischer, and Rajan (2000, 2001), we further assume that $S(\beta)$ is an increasing convex function on [0, 1] such that $S'(\beta) \geq 0$, $S''(\beta) \geq 0$, $S(0) = S'(0) = 0$ and $S(1) = S'(1) = +\infty$. These assumptions imply that the marginal cost of improving quality is increasing and it is impossible for the supplier to achieve a perfect quality level $\beta = 1$. The quality improvement cost includes staff training cost, machine maintenance cost and inspection cost, among others.

The manufacturer procures the components from the supplier and pays ω for each unit of delivered components. After a component is received by the manufacturer, it is inspected to test its quality. Such a test is called *incoming inspection*, see Baiman, Fischer, and Rajan (2000).We assume that the incoming inspection can correctly identify a component without defect, but will accept a defective component with a certain probability. The manufacturer decides incoming inspection quality l, which is defined as the probability that a defective component is detected. It costs the manufacturer $I(l)$ to achieve quality level l, where $l \in [0, 1]$, $I'(l) \geq 0$, $I''(l) \geq 0$, $I(0) = I'(0) = 0$, and $I(1) = I'(1) = +\infty$. If the component fails to pass the incoming inspection, it becomes an internal failure, and the supplier pays a price rebate d to the manufacturer as compensation. In addition, the supplier incurs a repair cost r to reprocess the defective component to satisfy the manufacturer's requirements. The reprocessed component is then resent to the manufacturer. The reprocessed component will pass the incoming inspection with probability 1.

Similarly, the manufacturer's quality level θ is defined as the probability that the manufacturing process is without defect. The manufacturer assembles the components. Then, the quality level θ refers solely to the quality of the assembly process regardless of the quality level of the components. That is, with a defective component, the manufacturing process may be failure-free which still results in a defective final product. It costs the manufacturer $M(\theta)$ to achieve the desired quality level θ, which has the same properties as those of the supplier's. We have $M'(\theta) \geq 0$, $M''(\theta) \geq 0$, $M(0) = M'(0) = 0$ for any $\theta \in [0, 1]$, and $M(1) = M'(1) = +\infty$. Note that if there is a quality test at the end of the manufacturing process, the quality level and cost can be internalised in θ and $M(\theta)$.

The final product is defective if any party in the supply chain fails to provide failure-free products, which is called 'the weakest link property' (see Baiman, Netessine, and Kunreuther (2003) and Chao, Iravani, and Savaskan (2009)). There are three ways to get a defective final product: (1) defective components and perfect manufacturing processing; (2) perfect components and defective manufacturing processing; (3) defective components and defective manufacturing processing. The manufacturer sells the final product to the market at unit price p. Although the failures caused by the supplier and the manufacturer are different, it makes no difference to the customer who reports that the product they have received is defective. Any defective product sold to the market will be reported by the customer, which incurs an external failure cost e. The external failure cost includes the cost of refunding or replacing the defective product, and the cost of rebuilding the manufacturer's reputation (see Baiman, Fischer, and Rajan (2000) and Balachandran and Radhakrishnan (2005)). We assume that the external failure cost remains constant regardless of who is responsible for the failure. In this paper, we show that the traditional fixed share rates contract cannot coordinate the supply chain to reach the best profit level. The partial allocation contract, proposed by this article, specifies the component purchase price ω and the external failure cost share rates which vary from case to case based on the information derived from failure root cause analysis. The manufacturer conducts failure root cause analysis to identify who should take responsibility for the external failure in the following manner: all components are inspected to test whether or not the supplier is at fault; if not, fault for the supplier can be excluded. In such a manner, the probability for the fault to the supplier includes the case that the supplier is at fault and the manufacturer is not and the case that both are at fault. Then, the probability for the fault to the manufacturer corresponds to the case that the root analysis excludes the fault for the supplier.

However, the failure root cause investigation is imperfect because it may erroneously attribute the manufacturer's fault to the supplier, and vice versa. Thus, we assume that the investigation identifies that the supplier takes responsibility for its own fault with probability q_s and that the supplier takes responsibility for the manufacturer's fault, by erroneous attribution, with probability q_m. We assume that $q_m \leq q_s$. Let f_s be the probability that the supplier takes responsibility for external failure and f_m be the probability that the manufacturer takes responsibility for external failure. Figure 1 shows a probability tree to facilitate the analysis. As shown in Figure 1, the final product is good with probability $[1 - (1 - \beta)(1 - l)]\theta$, the external failure is caused by the manufacturer with probability $[1 - (1 - \beta)(1 - l)](1 - \theta)$, while it is caused by the supplier with probability $(1 - \beta)(1 - l)$.

It follows that

$$f_s = q_s(1 - \beta)(1 - l) + q_m[1 - (1 - \beta)(1 - l)](1 - \theta), \tag{1}$$

$$f_m = (1 - q_s)(1 - \beta)(1 - l) + (1 - q_m)[1 - (1 - \beta)(1 - l)](1 - \theta). \tag{2}$$

Table 1 summarises the notation and terminologies.

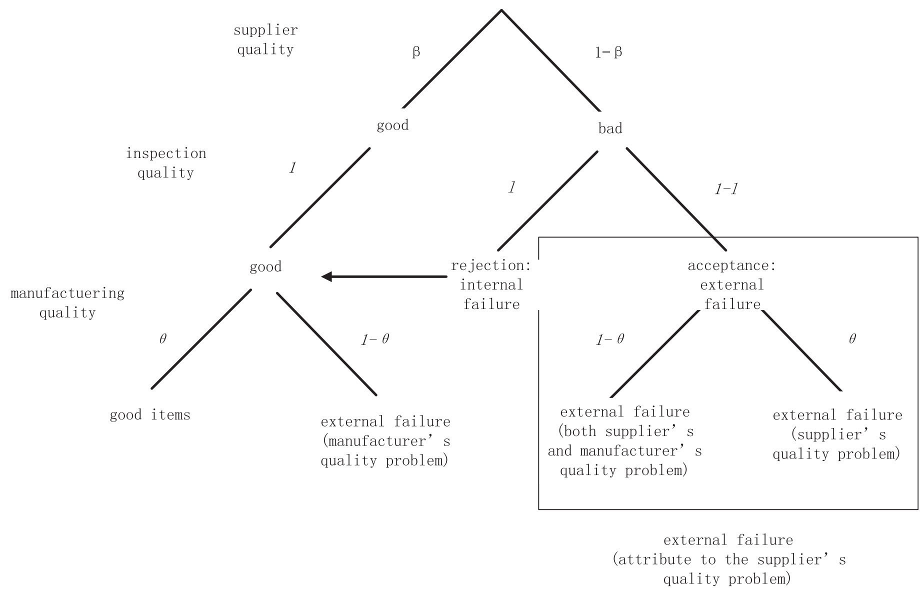

Figure 1. A probability tree of external failure.

Table 1. Basic notation.

β	The supplier's quality level
θ	The manufacturer's quality level
l	Quality level of the incoming inspection
$S(\beta)$	The supplier's quality cost
$M(\theta)$	The manufacturer's quality cost
$I(l)$	Exogenous investigation quality cost
c_s	The supplier's unit production cost
c_m	The manufacturer's unit production cost
ω	Unit purchase price
p	Unit sale price
d	Internal failure cost per unit component
e	External failure cost per unit product
r	Unit repair cost of the supplier for a component falling to pass incoming inspection
q_s	The probability that the supplier takes responsibility for its own fault
q_m	The probability that the supplier takes responsibility for the manufacturer's fault
f_s	The probability that the external failure is attributed to the supplier
f_m	The probability that the external failure is attributed to the manufacturer

4. Complete Information

We start by examining the complete information case where quality improvement efforts, together with the inspection quality level, are observable to the two members of the supply chain, and all other parameters in Table 1 are common knowledge. In the following, we show that the traditional fixed share rates contract, under which the external failure cost is shared between the supplier and the manufacturer at pre-determined fixed rates, cannot coordinate the supply chain to the centralised optimal quality levels. In contrast, the partial cost allocation contract can coordinate the supply chain to reach the centralised optimal quality levels, provided that the failure root cause analysis does not erroneously identify the manufacturer's fault as the supplier's.

4.1 *Centralised system*

We first study the centralised system, where a single firm decides the quality levels (β, θ, l) to maximise the total profit of the supply chain, as a benchmark. When the system is controlled by a single firm, the external cost is internalised.

The expected total profit of the supply chain is given by

$$\pi^C(\beta,\theta,l) = p - (1-\beta)lr - \{1-\theta[1-(1-\beta)(1-l)]\}e - S(\beta) - M(\theta) - I(l), \tag{3}$$

where $\theta[1-(1-\beta)(1-l)]e$ is the expected external failure cost.

We can verify that $\pi^C(\beta, \theta, l)$ is concave in β, θ and l, and so the optimal solutions for β, θ and l satisfy the first-order conditions as follows:

$$\frac{\partial \pi^C}{\partial \beta} = lr + \theta(1-l)e - S'(\beta) = 0, \tag{4}$$

$$\frac{\partial \pi^C}{\partial \theta} = [1-(1-\beta)(1-l)]e - M'(\theta) = 0, \tag{5}$$

$$\frac{\partial \pi^C}{\partial l} = -(1-\beta)r + \theta(1-\beta)e - I'(l) = 0. \tag{6}$$

Furthermore, we conclude that the profit function is concave in each parameter, and the strategy set is compact and concave by the fact that $\partial^2\pi^C/\partial\beta^2 \geq 0$, $\partial^2\pi^C/\partial\theta^2 \geq 0$, $\partial^2\pi^C/\partial l^2 \geq 0$, $0 \leq S'(\beta)$, $M'(\beta) \leq +\infty$, which also implies that there exists at least one optimal solution. We refer to β^*, θ^* and l^* as the centralised optimal quality levels, and $\pi(\beta^*, \theta^*, l^*)$ as the centralised optimal profit.

We compare the centralised optimal quality levels with the optimal quality levels in the decentralised system. In the decentralised system, the profit functions of the supplier and the manufacturer are, respectively, given by

$$\pi_s^D(\beta) = \omega - S(\beta) - (1-\beta)l(d+r); \tag{7}$$

$$\pi_m^D(\theta,l) = p - \omega - M(\theta) - I(l) + (1-\beta)ld - \{1-\theta[1-(1-\beta)(1-l)]\}e. \tag{8}$$

Then the supplier chooses its own quality level, and the manufacturer chooses its own quality level and its own incoming inspection quality level to maximise their own profit functions. Note that $\partial^2\pi_s^D(\beta)/\partial\beta^2 \leq 0, \partial^2\pi_m^D(\theta,l)/\partial^2\theta \leq 0, \partial^2\pi_m^D(\theta,l)/\partial l^2 \leq 0$, so the optimal quality levels in the decentralised system, denoted by$(\beta^{*D}, \theta^{*D}, l^{*D})$, satisfy the following first-order conditions:

$$\frac{\partial \pi_s^D(\beta)}{\partial \beta} = (d+r)l^{*D} - S'(\beta^{*D}) = 0, \tag{9}$$

$$\frac{\partial \pi_m^D(\theta,l)}{\partial \theta} = [1-(1-\beta^{*D})(1-l^{*D})]e - M'(\theta^{*D}) = 0, \tag{10}$$

$$\frac{\partial \pi_m^D(\theta,l)}{\partial l} = (1-\beta^{*D})d + \theta^{*D}(1-\beta^{*D})e - I'(l^{*D}) = 0. \tag{11}$$

It is easy to see that the equilibrium in the decentralised system, denoted by $(\beta^{*D}, \theta^{*D}, l^{*D})$, depends on the internal failure cost. By comparing (11) and (6), we find that there does not exist a d resulting in the centralised optimal quality levels, i.e. $(\beta^{*D}, \theta^{*D}, l^{*D}) = (\beta^*, \theta^*, l^*)$.Therefore, the decentralised system cannot achieve the centralised optimal quality levels.

4.2 *Partial cost allocation contract*

In this section, we adopt the partial cost allocation contract for the decentralised two-stage supply chain. The partial cost allocation contract is different from the fixed cost allocation contract, which allocates the external failure cost between the supplier and the manufacturer at a fixed share rate. It also differs from the single side cost allocation contract, under which the manufacturer receives the blame for all failures. The partial cost allocation contract allocates the external cost based on information derived from the failure root cause analysis. The share rates of the supplier and manufacturer vary from case to case depending on the failure root analysis. The manufacturer offers the supplier a partial cost allocation contract to coordinate the supply chain's quality levels. Specifically, the contract specifies the internal failure cost, and the share rates of the external cost between the supplier and manufacturer. Let $R_s(R_m)$ denote the supplier's share rate when the supplier (manufacturer) is at fault, where $0 \le R_s, R_m \le 1$. In particular, when $R_s = 1$ and $R_m = 0$, it reduces to the case where the one who is responsible for the failure bears the external failure cost. When $R_s = R_m$, it reduces to the fixed share rates contract, under which the external failure cost is allocated between the supplier and the manufacturer at pre-determined fixed rates.

The profit functions of the supplier and the manufacturer are, respectively, given by

$$\pi_s^P(\beta) = \omega - S(\beta) - (1-\beta)l(d+r) - (R_s f_s + R_m f_m)e, \tag{12}$$

$$\pi_m^P(\theta, l) = p - \omega - M(\theta) - I(l) + (1-\beta)ld - \{(1-R_s)f_s + (1-R_m)f_m\}e. \tag{13}$$

It is evident that $\partial^2\pi_s^P/\partial\beta^2 \le 0, \partial^2\pi_m^P/\partial\theta^2 \le 0, \partial^2\pi_m^P/\partial l^2 \le 0$, and the strategy space is compact and convex, which guarantees the existence of at least one pure strategy Nash equilibrium. Given the share rates, the best responses of the supplier and manufacturer satisfy the following first-order conditions:

$$\frac{\partial\pi_s^P}{\partial\beta} = l(d+r) - [-q_s(1-l) + q_m(1-l)(1-\theta)]R_s e - [-(1-q_s)(1-l) + (1-q_m)(1-l)(1-\theta)]R_m e - S'(\beta) = 0, \tag{14}$$

$$\frac{\partial\pi_m^P}{\partial\theta} = q_m[1-(1-\beta)(1-l)](1-R_s)e + (1-q_m)[1-(1-\beta)(1-l)](1-R_m)e - M'(\theta) = 0. \tag{15}$$

$$\frac{\partial\pi_m^P}{\partial l} = (1-\beta)d - [-q_s(1-\beta) + q_m(1-\beta)(1-\theta)](1-R_s)e - [-(1-q_s)(1-\beta) + (1-q_m)(1-\beta)(1-\theta)] (1-R_m)e - I'(l) = 0. \tag{16}$$

If the partial cost allocation contract coordinates the supply chain to reach the best profit level, then (β^*, θ^*, l^*) must be a solution to (14)–(16). Substituting (β^*, θ^*, l^*) into (14)–(16), we obtain the following Proposition.

Proposition 1. The partial cost allocation contract coordinates the supply chain to reach the centralised optimal profit level only when $q_m = 0$. The optimal external failure cost share rates and internal failure cost are, respectively, given by

$$R_m^{*P} = 0, R_s^{*P} = \frac{(1-l^*)\theta^* e + rl^*}{q_s e}, d^{*P} = (1-l^*)(\theta^* e - r). \tag{17}$$

Proof. The proof is given in the Appendix.

Proposition 1 shows that if the exogenous investigation does not erroneously identify the manufacturer's fault as the supplier's, the partial cost allocation contract can achieve the best profit level for the supply chain. To achieve the centralised optimal profit, the coordinating contract requires that the manufacturer takes responsibility for its own fault and thus the supplier need not share the external failure cost caused by the manufacturer. Proposition 1 also yields two implications as follows:

(1) In the optimal solution, $R_m^{*P} \ne R_s^{*P}$, which implies that the fixed cost allocation contract, which is a special case of the partial cost allocation contract when $R_m = R_s$, cannot coordinate the supply chain to reach the centralised optimal profit level.
(2) In the optimal solution, $R_s^{*P} \ne 1$, which implies that the contract specifying that the supplier bears the internal failure cost and the manufacturer bears the external failure cost does not coordinate the supply chain to achieve the centralised optimal profit.

In summary, to achieve the centralised optimal profit level for the supply chain, the external failure cost should be shared at positive rates between the supplier and manufacturer. To ensure the manufacturer has an incentive to achieve its desired centralised optimal quality levels $\theta^*$and l^*, the supplier needs to share R_s^{*P} of the external failure cost for its own fault causing the external failure. To ensure the supplier has an incentive to achieve its desired centralised optimal quality level β^*, the manufacturer needs to share $1 - R_s^{*P}$ of the external failure cost for the supplier's fault which causes the external failure, and must bear the entire external failure cost caused by its own fault.

5. Moral hazard models

For the moral hazard models where the quality levels β and θ are not observable (completely or partially), we consider both the single moral hazard model and the double moral hazard model to determining whether or not the partial cost allocation contract can achieve the centralised optimal profit.

5.1 *Single moral hazard*

In the single moral hazard model, the supplier's quality level β is unobservable to the manufacturer. The manufacturer first decides quality level θ and the quality level of incoming inspection l, then it conveys quality levels θ and l to the supplier. Then the manufacturer offers the supplier a contract specifying d, R_s and R_m to induce the supplier to choose the optimal quality level that maximises the manufacturer's profit. The supplier decides whether or not to accept the contract. The contract should satisfy two constraints from the supplier's perspective, (i) the participation constraint or individual rationality constraint (IR) and (ii) the incentive compatibility constraint (IC). After observing the manufacturer's quality levels, the supplier decides whether or not to accept the contract. The supplier chooses its own quality level β to maximise the profit. If the supplier's profit is nonnegative, then it accepts the contract; otherwise, the contract is rejected. If the supplier refuses to accept the contract, nothing is received.

The manufacturer's profit maximisation problem can be formulated mathematically as follows:

$$\max_{l,\theta,R_s,R_m} \quad \pi_m^P(\beta,\theta,l,d,R_s,R_m) \tag{18}$$

$$\text{s.t.} \quad (\text{IR}) \quad \pi_s^P(\beta,\theta,l,d,R_s,R_m) \geq 0, \tag{19}$$

$$(\text{IC}) \quad \beta = arg\max_{\beta} \pi_s^P(\beta,\theta,l,d,R_s,R_m), \tag{20}$$

$$0 \leq \beta,\theta,l,R_s,R_m \leq 1, \omega \geq 0. \tag{21}$$

The IR constraint means that the supplier should have an incentive to accept the contract, i.e. the supplier's payoff should be greater than its reservation profit if it is to accept the contract. Changing the reservation profit to any positive value does not affect our analysis. The IC constraint ensures the supplier has an incentive to choose the quality level specified by the manufacturer, i.e. the supplier can maximise its own profit by choosing the quality level.

Proposition 2. In the single moral hazard model, the centralised optimal quality levels are achieved, i.e. $\beta^{*S} = \beta^*$, $\theta^{*S} = \theta^*$, and $l^{*S} = l^*$. The optimal share rates, denoted by R_s^{*S} and R_m^{*S} are, respectively, given by

$$R_s^{*S} = \frac{[S'(\beta^*) - (d+r)l^*]f_m - (1-l^*)[q_m - q_s + (1-q_m)\theta^*][\omega - S(\beta^*) - (1-\beta^*)(d+r)l^*]}{(q_s - q_m)(1-\theta^*)(1-l^*)e}, \tag{22}$$

$$R_m^{*S} = \frac{-[S'(\beta^*) - (d+r)l^*]f_s - (1-l^*)(q_m - q_s - q_m\theta^*)[\omega - S(\beta^*) - (1-\beta^*)(d+r)l^*]}{(q_s - q_m)(1-\theta^*)(1-l^*)e}, \tag{23}$$

where d should be appropriately chosen to ensure that $0 \leq R_s, R_m \leq 1$. Specifically, $d \in [\underline{d}, \bar{d}]$, where $\bar{d} = \frac{S'(\beta^*)[1-\theta^*(1-(1-\beta^*)(1-l^*))] - \theta^*(1-l^*)(\omega - S(\beta^*))}{l^*(1-\theta^*)} - r$ and $\underline{d} = \max\left\{\frac{S'(\beta^*)f_s + (1-l^*)(q_m - q_s - \theta^* q_m)(\omega - S(\beta^*))}{q_m l^*(1-\theta^*)} - r, \frac{S'(\beta^*)f_m - (q_s - q_m)(1-\theta^*)(1-l^*)e - (1-l^*)(q_m - q_s - \theta^* q_m + \theta^*)(\omega - S(\beta^*))}{l^*(1-q_m)(1-\theta^*)} - r\right\}$.

Proposition 2 establishes that the partial cost allocation contract coordinates the supply chain to reach the best profit level. Since the supplier only obtains a profit of zero, the manufacturer receives the entire supply chain profit when the supplier does not report its quality level.

5.2 *Double moral hazard*

In this section, we consider the case where the manufacturer's quality level θ is not observable to the supplier and supplier's quality level β is not observable to the manufacturer. The manufacturer offers the supplier a contract, which specifies the wholesale price ω, the price rebate d, and the share rates R_s^D and R_m^D. The contract aims to induce the supplier to choose the quality level that maximising the manufacturer's profit. Similarly to the single moral hazard case, the double moral hazard model is subject to the supplier's individual rationality constraint and incentive compatibility constraint, in addition to the manufacturer's incentive constraint. The optimisation problem associated with the double moral hazard model can be formulated mathematically as follows:

$$
\begin{array}{lll}
\max\limits_{l,\omega,d,R_s,R_m} & & \pi_m^P(\beta,\theta,l,d,\omega,R_s,R_m) \\
\text{s.t.} & \text{(IR)} & \pi_s^P(\beta,\theta,l,d,\omega,R_s,R_m) \geq 0, \\
 & \text{(IC)} & \beta = arg\max\limits_{\beta} \pi_s^P(\beta,\theta,\omega,d,R_s,R_m), \\
 & \text{(IC}'\text{)} & \theta = arg\max\limits_{\theta} \pi_m^P(\beta,\theta,l,d,\omega,R_s,R_m), \\
 & & 0 \leq \beta,\theta,R_s,R_m \leq 1, \omega, d \geq 0.
\end{array} \tag{24}
$$

Proposition 3. In the double moral hazard model, the centralised optimal quality levels are achieved provided that the failure root analysis does not erroneously identify the manufacturer's fault as the supplier's. In other words, $\beta^{*D} = \beta^*$, $\theta^{*D} = \theta^*$ and $l^{*D} = l^*$ only when $q_m = 0$. The share rates and the wholesale price are, respectively, given by $R_m^{*D} = 0, R_s^{*D} = \frac{\theta^*(1-l^*)e-d^{*D}l^*}{q_s(1-l^*)e}$, and $\omega^{*D} = S(\beta^*) + (1-\beta^*)S'(\beta^*)$, where d^{*D} satisfies $\frac{(1-l^*)(\theta^*e-q_se)}{l^*} \leq d^{*D} \leq \frac{\theta^*(1-l^*)e}{l^*}$. However, if the failure root analysis erroneously identifies the manufacturer's fault as the supplier's, i.e. $q_m \neq 0$, then the centralised optimal quality levels cannot be achieved.

Proposition 3 establishes that in the double moral hazard model the centralised optimal solution is achieved only when the failure root cause analysis can correctly identify the manufacturer's fault. In this case, the manufacturer bears the entire external failure cost when it is at fault and shares $1 - R_s^{*D}$ of the external cost when the supplier is at fault. The share rate R_s^{*D} is decreasing in q_s, i.e. the share rate of the supplier increases as the probability that the failure root cause analysis correctly identifies the supplier's fault decreases.

5.3 *Degenerated contracts*

The partial cost allocation contract can degenerate into two commonly used contracts. One is the single side cost allocation contract, which specifies that the manufacturer takes full responsibility for its own fault and shares the external failure cost when the supplier is at fault. The other is the fixed cost allocation contract, under which the external failure cost is shared between the manufacturer and the supplier at fixed share rates. As shown in Section 4, the fixed share rates contract cannot coordinate the quality improvement efforts and achieve the centralised optimal quality levels in the complete information case. The following lemmas present the optimal solutions for the degenerated contracts under the incomplete information cases.

Lemma 1. (i) In the single moral hazard model, the single side cost allocation contract achieves the centralised optimal quality levels. The optimal internal penalty cost d^{*F_1} and the share rate R^{*F_1} are given by $R^{*F_1} = \frac{\omega-S(\beta^*)-(1-\beta^*)S'(\beta^*)}{q_m(1-\theta^*)e}$ and $d^{*F_1} = \frac{S'(\beta)[q_m(1-\theta^*)(l+\beta^*-l^*\beta^*)+q_s(1-\beta^*)(1-l^*)]-(1-l^*)(q_s-q_m+q_m\theta^*)(\omega-S(\beta^*))}{q_m(1-\theta^*)l^*} - r$. ω should be bounded in $[S(\beta^*) + S'(\beta^*)(1-\beta^*), S(\beta^*) + S'(\beta^*)(1-\beta^*) + q_me(1-\theta^*)]$.

(ii) In the double moral hazard model, the single side cost allocation coordinates the quality levels and the optimal contract setting is the same as that given in Proposition 3.

Proof. It is easy to find that the partial cost allocation contract degenerates into the single side cost allocation contract when $R_\mathrm{m} = 0$. Therefore, by setting R_m to zero in Propositions 2 and 3, we establish Lemma 1.

Lemma 2. (i) In the single moral hazard model, the fixed cost allocation contract coordinates the quality levels. Furthermore, $d^{*F_2} = \frac{S'(\beta^*)[1-\theta^*l-\theta^*\beta^*+\theta^*\beta^*l^*]-\theta^*(1-l^*)[\omega-S(\beta^*)]}{l^*(1-\theta^*)} - r$ and $R^{*F_2} = \frac{\omega-S(\beta^*)-S'(\beta^*)(1-\beta^*)}{(1-\theta^*)e}$, where ω should be bounded in $[S(\beta^*) + S'(\beta^*)(1-\beta^*), (1-\theta^*)e + S(\beta^*) + S'(\beta^*)(1-\beta^*)]$.

(ii) In the double moral hazard model, the fixed cost allocation contract coordinates the quality levels only when $q_m = 0$. Furthermore, $R^{*F_2} = 0, d^{*F_2} = \frac{\theta e(1-l)}{l}$, and $\omega^{*F_2} = S(\beta) + (1-\beta)S'(\beta)$.

Proof. The proof is similar to that of Lemma 1 by setting $R_s = R_m$.

Lemmas 1 and 2 show that the single side cost allocation contract and the fixed cost allocation contract are special cases of the partial cost allocation contract. The degenerated contracts require that the manufacturer has the power to determine the internal failure cost. But the partial cost allocation contract is much more flexible as illustrated in the numerical examples in the following comparison.

5.4 *Comparison with partial cost allocation contract between degenerated contracts*

In the following content, numerical examples are presented to show the flexibility and fairness of the partial cost allocation contract by comparing it with the single side cost allocation contract and the fixed cost allocation contract. The numerical examples below focus on the single moral hazard model to show the effectiveness of the partial cost allocation contract. We assume that all of the quality costs have the logarithmic form, i.e. $S(\beta) = s\ln\frac{1}{1-\beta}, M(\theta) = m\ln\frac{1}{1-\theta}$, and $I(l) = a\ln\frac{1}{1-l}$. The logarithmic form ensures that the first derivatives of these cost functions at point 1 are equal to infinity. Solving (4)–(6), we obtain the centralised optimal solution as follows:

$$\theta_i^* = \frac{1}{2e^2}\left\{-[(m-r-a)e-e^2] \pm \sqrt{[(m-r-a)e-e^2]^2 - 4e^2(re+ae-mr)}\right\}, \quad i = 1,2,$$
$$\beta^* = 1 - \frac{s-a}{r}, \quad l_i^* = 1 - \frac{a+e-m-\theta_i^* e}{(1-\beta^*)(e-r)}.$$

We note that there exist two solutions to the first-order conditions (4)–(6), i.e. $x_1 = (\beta^*, \theta_1^*, l_1^*)$ and $x_2 = (\beta^*, \theta_2^*, l_2^*)$. The centralised optimal solution should be the one that is bounded in the space $\Omega = [0, 1] \otimes [0, 1] \otimes [0, 1]$. If $x_i \in \Omega$ for each i, $i = 1, 2$, then the centralised optimal solution x^* is the one that minimises the corresponding cost, i.e. $x^* = \min_{i=1,2}\{1-\theta_i^*[1-(1-\beta^*)(1-l_i^*)]\}e + M(\theta_i^*) + I(l_i^*)\}$. Noting that these two solutions have the same value of β, there is no need to take the cost related to β into consideration when we choose the centralised optimal solution. In addition, we assume that $0 < s - a < r$, where the first inequality indicates that the marginal cost of improving the quality level of the supplier is much higher than that of the incoming inspection, while the second inequality indicates that the supplier has an incentive to improve its own quality level.

Balachandran and Radhakrishnan (2005) propose that a contract should satisfy a fairness criterion to comply with the law; otherwise, the contract will be considered invalid in court. The authors define as fairness criterion that the penalty cost passed onto the supplier should not be more than the external failure cost that the manufacturer incurs. However, in our case, the penalty cost passed onto the supplier includes not only the external failure cost but also the internal failure cost. Therefore, we define as fairness criterion in this paper that the internal failure penalty and the external failure penalty that the supplier incurs should not be greater than the external failure cost that the manufacturer incurs. Mathematically, the fairness criterion should satisfy the following inequality:

$$dif^C = [(1-R_s^C)f_s + (1-R_m^C)f_m]e - (1-\beta)ld - (R_s^C f_s + R_m^C f_m)e \geq 0, \tag{25}$$

where dif^C denote difference between the failure cost attributed to the manufacture and the total penalty cost (internal and external cost) attributed to the supplier. Meanwhile, the superscripts with $C = F_1, F_2$ denote the cases for single side cost allocation contract and the fixed cost allocation contract, respectively.

The numerical results reported in Table 2 show that the fairness criterion may not always be satisfied for degenerated contracts. We find that, although the centralised optimal solution is achieved under each contract, the fairness criterion may not be satisfied under the fixed share rates contract. In comparison, the fairness criterion is satisfied under the partial cost allocation contract by properly choosing (d, ω). From the perspective of the fairness criterion, it appears that the partial cost allocation contract is easier to implement than the fixed share rates contract, even though the latter has a simpler structure. It is noted that the case without solution indicates that the corresponding contract cannot coordinate the supply chain into first best profit.

Table 2. Contract comparison with $s = 3$, $m = 2$, $a = 1$, $e = 20$, $r = 4$.

Instance (q_s q_m)	ω	d^{*C}	dif^C	d^{*F1}	dif^{F1}	d^{*F2}	dif^{F2}
(0.9 0.3)	7.0	0.171	−2.901	–	–	0.329	−4.545
(0.9 0.3)	5.5	1.761	0.781	1.108	−1.212	2.425	−0.648
(0.8 0.3)	7.0	0.197	−2.890	–	–	0.329	−4.545
(0.8 0.3)	5.5	1.876	0.828	1.328	−1.118	2.425	−0.648
(0.7 0.3)	5.5	1.986	0.875	1.547	−1.024	2.425	−0.648
(0.6 0.3)	5.5	2.096	0.922	1.767	−0.930	2.425	−0.648
(0.5 0.3)	5.5	2.206	0.969	1.986	−0.836	2.425	−0.648
(0.7 0.1)	5.5	1.519	0.676	–	–	2.425	−0.648
(0.7 0.2)	5.5	1.602	0.711	0.778	−1.353	2.425	−0.648
(0.7 0.3)	5.5	1.986	0.875	1.547	−1.024	2.425	−0.648
(0.7 0.4)	5.5	2.178	0.957	1.931	−0.859	2.425	−0.648
(0.7 0.5)	5.5	2.294	1.007	6.157	−0.761	2.425	−0.648

6. Partial cost allocation contract with dual root cause analyses

6.1 *Optimal share rates under dual root cause analyses*

In Section 5, we have considered the partial cost allocation contract for both single moral hazard and double moral hazard, and proved that the partial cost allocation contract is more flexible than the single side cost allocation contract as well as the fixed cost allocation contract. In this section, we discuss the manufacturer's advantage on the root cause analysis, and propose a partial cost allocation contract with dual root analyses to reduce the penalty cost of the supplier.

Recall that in the partial cost allocation contract with the root cause analysis conducted by the manufacturer, the external failure is attributed to the supplier when both players are at fault. Under this contract, even though the supplier's quality level is higher than the manufacturer's, the supplier may have higher probability of being blamed for the external failure than the manufacturer. It is unfair to the supplier. Based on this observation, we propose another partial cost allocation contract with dual root cause analyses to reduce the external failure cost attributed to the supplier due to manufacturer's advantage.

When an external failure occurs, the partial cost allocation contract with dual root cause analyses is conducted in the following manners:

(1) The contract specifies that a fraction α of the external failure will be analysed by the root cause analysis conducted by the manufacturer. The manufacturer conducts the root cause analysis on the components of supplier. When the supplier is not at fault, then external fault is attributed to the manufacturer (the probability that the external failure attributed to the supplier is the sum of the probability that the supplier is at fault but the manufacturer is not, and that both are at fault). The manufacturer proposes partial allocation rates R_m^*, R_s^*, where the external failure cost attributed to the supplier is $(R_s^{*S} f_s + R_m^{*S} f_m)e$.

(2) The contract specifies that the remaining fraction $1-\alpha$ of the external failure will be analysed by the root cause analysis conducted by the supplier. The supplier conducts a root cause analysis on the manufacturing process. When the manufacturing process is without defection, the fault is attributed to the supplier (the probability that the external failure is attributed to the manufacturer is the sum of the probability that the manufacturer is at fault but the supplier is not, and both are at fault). The supplier proposes partial allocation rates $\tilde{R}_s^{*S}$ and $\tilde{R}_m^{*S}$, where the external failure cost attributed to the supplier is $(\tilde{R}_s^{*S} f_s + \tilde{R}_m^{*S} f_m)e$.

Next, we develop the optimal partial allocation scheme $\tilde{R}_s^{*S}$ and $\tilde{R}_m^{*S}$ when the supplier conducts the root cause analysis, which also leads to centralised optimal quality levels.

Under supplier's root cause analysis, the external failure is attributed to the manufacturer with probability$1-\theta$ and to the supplier with probability $(1-\beta)(1-l)\theta$. It follows that

$$\tilde{f}_s = q_s(1-\beta)(1-l)\theta + q_m(1-\theta),$$

$$\tilde{f}_m = (1-q_s)(1-\beta)(1-l)\theta + (1-q_m)(1-\theta).$$

The probability tree is given in Figure 2.

Following a similar argument to Propositions 2 and 3, we get Propositions 4 and 5 as follows:

Proposition 4. In the single moral hazard model when the supplier performs the root cause analysis, the centralised optimal quality levels are achieved. The optimal share rate, denoted by $\tilde{R}_s^{*S}$ and $\tilde{R}_m^{*S}$, are given by

$$\tilde{R}_s^{*S} = \frac{(1-q_m)(1-\theta^*)[S'(\beta^*) - (d+r)l^*] - (1-q_s)(1-l^*)\theta^*[\omega - S(\beta^*) - (1-\beta^*)S'(\beta^*)]}{(q_s-q_m)(1-l^*)(1-\theta^*)\theta^* e},$$

$$\tilde{R}_m^{*S} = \frac{-q_m(1-\theta^*)[S'(\beta^*) - (d+r)l^*] + q_s(1-l^*)\theta^*[\omega - S(\beta^*) - (1-\beta^*)S'(\beta^*)]}{(q_s-q_m)(1-l^*)(1-\theta^*)\theta^* e},$$

where d should be bounded in $[\underline{d'}, \overline{d'}]$, with

$$\underline{d'} = \max\Big\{\frac{q_m(1-\theta^*)S'(\beta^*) - q_s(1-l^*)\theta^*[\omega - S(\beta^*) - (1-\beta^*)S'(\beta^*)]}{q_m(1-\theta^*)l^*} - r,$$
$$\frac{(1-q_m)(1-\theta^*)S'(\beta^*) - (q_s-q_m)(1-l^*)(1-\theta^*)e - (1-q_s)(1-l^*)\theta^*[\omega - S(\beta^*) - (1-\beta^*)S'(\beta^*)]}{q_m(1-\theta^*)l^*} - r\Big\},$$

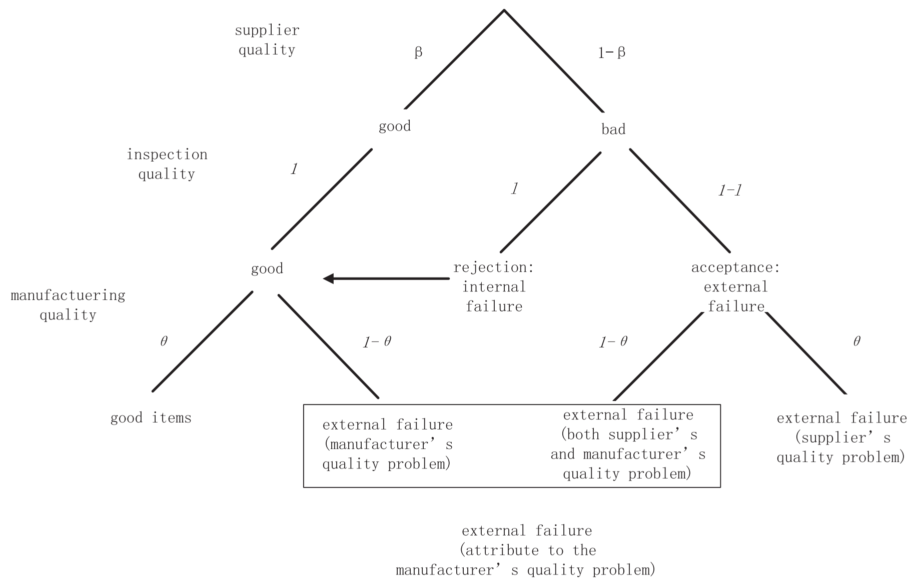

Figure 2. The probability tree when the supplier conducts the root cause analysis.

$$\overline{d'} = \min\{\frac{(1-q_m)(1-\theta^*)S'(\beta^*) - (1-q_s)(1-l^*)\theta^*[\omega - S(\beta^*) - (1-\beta^*)S'(\beta^*)]}{(1-q_m)(1-\theta^*)l^*} - r,$$
$$\frac{q_m(1-\theta^*)S'(\beta^*) + (q_s-q_m)(1-l^*)(1-\theta^*)e - q_s(1-l^*)\theta^*[\omega - S(\beta^*) - (1-\beta^*)S'(\beta^*)]}{q_m(1-\theta^*)l^*} - r\}.$$

Next, we show that $[\underline{d}, \bar{d}]$ and $[\underline{d}', \overline{d'}]$ has intersection, as follows:

$$\underline{d}' \le \frac{q_m(1-\theta^*)S'(\beta^*) - q_s(1-l^*)\theta^*[\omega - S(\beta^*) - (1-\beta^*)S'(\beta^*)]}{q_m(1-\theta^*)l^*} \le \frac{q_m(1-\theta^*)S'(\beta^*) - q_m(1-l^*)\theta^*[\omega - S(\beta^*) - (1-\beta^*)S'(\beta^*)]}{q_m(1-\theta^*)l^*}$$
$$= \frac{(1-\theta^*)S'(\beta^*) - (1-l^*)\theta^*[\omega - S(\beta^*) - (1-\beta^*)S'(\beta^*)]}{(1-\theta^*)l^*} = \frac{(1-\theta^*)S'(\beta^*) - (1-l^*)\theta^*[\omega - S(\beta^*)] + (1-l^*)\theta^*(1-\beta^*)S'(\beta^*)]}{(1-\theta^*)l^*}$$
$$= \frac{(1-\theta^* + (1-l^*)\theta^*(1-\beta^*))S'(\beta^*) - (1-l^*)\theta^*[\omega - S(\beta^*)]}{(1-\theta^*)l^*} = \underline{d}.$$

(The second inequality hold by the assumption that $q_m \le q_s$), which means there exists a d such that it coordinates the supply chain in both cases no matter who conducts the root cause analysis, the manufacturer or the supplier.

From Propositions 2 and 4, we know that R_s^{*S} and R_m^{*S} coordinate the supply chain to achieve the centralised optimal quality levels when the manufacturer takes the root cause analysis, and $\tilde{R}_s^{*S}$ and $\tilde{R}_m^{*S}$ coordinate the supply chain when the supplier takes the root cause analysis. Meanwhile, since the two parts of products achieve their centralised optimal profits, then the whole supply chain also achieves its centralised optimal profit. Then, one can see that on average the external failure costs of the supplier's and the manufacturer, denoted by $\overline{EC}_i, i = s, m$, are equal to,

$$\overline{EC}_s = \alpha(R_s^{*S} f_s + R_m^{*S} f_m)e + (1-\alpha)(\tilde{R}_s^{*S} f_s + \tilde{R}_m^{*S} f_m)e,$$
$$\overline{EC}_m = \alpha[(1-R_s^{*S})f_s + (1-R_m^{*S})f_m]e + (1-\alpha)[(1-\tilde{R}_s^{*S})f_s + (1-\tilde{R}_m^{*S})f_m]e.$$

Proposition 5. In the double moral hazard model when the supplier performs the root cause analysis, the centralised optimal quality levels are achieved provided that $q_m = q_s(1-\beta^*)(1-l^*)$. Then the share rates are given by $\tilde{R}_m^{*D} = 0, \tilde{R}_s^{*D} = \frac{S'(\beta^*) - (d+r)l^*}{q_s(1-l^*)\theta^* e}$, and $\tilde{\omega}^{*D} = \frac{(1-\beta^*)[S'(\beta^*) - (d+r)l^*]}{\theta^*} + S(\beta^*) + (1-\beta^*)(d+r)l^*$, where d satisfies $\frac{S'(\beta) - q_s(1-l)\theta e}{l} - r \le d \le \frac{S'(\beta)}{l} - r$.

From Propositions 3 and 5, we obtain that when the root cause analysis is conducted by the manufacturer, R_s^{*D} and R_m^{*D} coordinate the supply chain to achieve the centralised optimal quality levels provided that q_m=0. When the root cause analysis is conducted by the supplier, $\tilde{R}_s^{*D}$ and $\tilde{R}_m^{*D}$ coordinate the supply chain to achieve the centralised optimal

Table 3. Contract comparison with s = 3, m = 2, a = 1, e = 20, r = 4.

Instance (q_s q_m)	ω	d	R_s^{*S}	R_m^{*S}	$\tilde{R}_s^{*S}$	$\tilde{R}_m^{*S}$	dif^S
(0.5 0.3)	5.4	2.398	0.322	0.074	0.343	0.065	0.014
(0.6 0.3)	5.4	2.314	0.322	0.074	0.343	0.065	0.012
(0.7 0.3)	5.4	2.230	0.322	0.074	0.343	0.065	0.010
(0.8 0.3)	5.4	2.147	0.322	0.074	0.343	0.065	0.008
(0.9 0.3)	5.4	2.063	0.322	0.074	0.343	0.065	0.0054
(0.7 0.4)	5.4	2.377	0.260	0.074	0.274	0.066	0.006
(0.7 0.5)	5.4	2.465	0.223	0.074	0.232	0.065	0.003
(0.7 0.6)	5.4	2.523	0.198	0.074	0.204	0.065	0.001
(0.7 0.6)	5.3	2.676	0.136	0.051	0.141	0.045	0.004

quality levels provided that $q_m = q_s(1-\beta^*)(1-l^*)$. As $q_m \to 0$, the average external failure costs of the supplier and the manufacturer, denoted by $\overline{EC}'_i, i = s, m$, are given as

$$\overline{EC}'_s = \alpha(R_s^{*D}f_s + R_m^{*D}f_m)e + (1-\alpha)(\tilde{R}_s^{*D}f_s + \tilde{R}_m^{*D}f_m)e,$$
$$\overline{EC}'_m = \alpha[(1-R_s^{*D})f_s + (1-R_m^{*D})f_m]e + (1-\alpha)[(1-\tilde{R}_s^{*D})f_s + (1-\tilde{R}_m^{*D})f_m]e.$$

6.2 *Comparison between dual root cause analyses and single root cause analysis*

We numerically show the fairness criterion under the contract with the dual root cause analyses and single root cause analysis in Table 3. A special case $\alpha = 0.5$ for the dual root cause analyses is considered, i.e. half of the external failure is supposed to be analysed by the manufacturer while the remaining half is supposed to be done by the supplier. The fairness criterion can be illustrated through dif^S, where

$$dif^S = (R_s^{*S}f_s + R_m^{*S}f_m)e - \overline{EC}'_s = \frac{(R_s^{*S}f_s + R_m^{*S}f_m)e - (\tilde{R}_s^{*S}f_s + \tilde{R}_m^{*S}f_m)e}{2}$$

represents the penalty cost difference that the supplier should take in the single root cause analysis case and in the dual root cause analyses case, in which the supplier's penalty cost includes its internal failure cost and its external cost and the internal costs for the two contracts are the same. It is noted that $dif^S > 0$ denotes that the supplier pays a smaller penalty cost under the dual root cause analyses contract than that under the single root cause analysis contract. From Table 3, we see that the supplier can reduce its penalty cost by choosing the contract based on the information from the dual root cause analyses.

7. Conclusions

In this paper, we consider quality improvement efforts coordination and incentives for a supply chain with failure root analysis. The external failure cost is shared between the manufacturer and the supplier at different share rates which vary from case to case, depending on who is responsible for the external failure. We show that the partial cost allocation contract coordinates the quality improvement efforts of the supply chain members and achieves the centralised optimal profit when there is no failure root cause analysis cost. We assume that demand is exogenous and deterministic. However, in some cases, demand is sensitive to product quality, and so considering quality improvement efforts coordination and incentives for a supply chain with quality-sensitive demand is an interesting future research direction. One may also study how the structure of quality-sensitive demand affects the coordination of the quality improvement efforts of supply chain members. We use root cause analysis to generate the probability of who is responsible for the external failure, but the Bayes rule provides another alternative way to form this probability. Is Bayes rule an effective tool to coordinate the quality levels? Who would benefit from the Bayes rule, supplier or manufacturer? These questions inspire our future research interest.

Furthermore, this paper uses a penalty contract to coordinate the supply chain to implement centralised optimal quality levels; however, is there a revenue-sharing contract which can coordinate the supply chain? This is another worthy direction for future research.

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Note

1. Data from CAAS, 2011 China Automobile and After-sale Service Quality Report: http://www.caq.org.cn/html/cse_result/2012–3/4/221647.shtm.

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Appendix 1

Proof of Proposition 1

If the partial cost allocation contract coordinates the quality improvement efforts of supply chain members and achieves the centralised optimal quality levels, we substitute (β^*, θ^*, l^*) into (14)–(16) and compare the results with (4)–(6) to get

$$\begin{aligned} ld - [-q_s(1-l^*) + q_m(1-l^*)(1-\theta^*)]R_s e - [-(1-q_s)(1-l^*) \\ +(1-q_m)(1-l^*)(1-\theta^*)]R_m e = \theta^*(1-l^*)e \end{aligned} \tag{26}$$

$$q_m R_s + (1 - q_m) R_m = 0, \tag{27}$$

$$[-q_s + q_m(1-\theta^*)]R_s e + [-(1-q_s) + (1-q_m)(1-\theta^*)]R_m e = -r - d. \tag{28}$$

In order to solve (26)–(28), we distinguish three cases as follows:

Case 1: If $0 < q_m < 1$, then we have $R_m^{*P} = R_s^{*P} = 0$ from (27). But substituting $R_m^{*P} = R_s^{*P} = 0$ into (26) and (28), we find that this case is not a solution to the equations.

Case 2: If $q_m = 1$, then we have $R_s = 0$. Substituting $q_m = 1$ and $R_s^{*P} = 0$ into (26) and (28), we get $d^{*P} = (1 - l^*)(\theta^* e - r)$ and $R_m^{*P} = \frac{(1-l^*)\theta^* e + rl^*}{(1-q_s)e}$. But this is a feasible coordinating contract. Under the partial cost allocation contract, we assume that $R_s \geq R_m$, which means that the supplier should take more responsibility for its own fault than that for the manufacturer's fault. But this solution does not satisfy this assumption.

Case 3: If $q_m = 0$, then we have $R_m = 0$ from (27). Substituting $q_m = 0$ and $R_m^{*P} = 0$ into (26) and (28), we get $d^{*P} = (1 - l^*)(\theta^* e - r)$ and $R_s^{*P} = \frac{(1-l^*)\theta^* e + rl^*}{q_s e}$.

Proof of Proposition 2

Note that in the optimisation problem (24), π_s^P is concave in β. Then the IC constraint can be rewritten in the form of the first-order condition as given in (20). Furthermore, the IR constraint must be binding. Otherwise the manufacturer's profit can be improved by increasing the price rebate d, or increasing R_s and R_m, while the constraints still hold. Therefore, the optimisation problem can be expressed as

$$\begin{array}{ll} \max\limits_{\beta,\theta,l,d,R_s,R_m} & \pi_m^P(\beta,\theta,l,\omega,R_s,R_m) \\ \text{s.t.} \quad (\text{IR}) & \pi_s^P(\beta,\theta,l,d,R_s,R_m) = 0, \\ \qquad\ (\text{IC}) & \partial\pi_s^P(\beta,\theta,l,d,R_s,R_m)/\partial\beta = 0, \\ & 0 \leq \beta,\theta,l,R_s,R_m \leq 1, d \geq 0. \end{array} \tag{29}$$

The above constrained optimisation problem can be solved by the method of Lagrange multipliers, which yields a necessary condition for optimality. The Lagrange function F is given by

$$\begin{aligned} F = p + (1-\beta)ld - \omega - \{(1-R_s)f_s + (1-R_m)f_m\}e - M(\theta) - I(l) + \lambda\{\omega - (1-\beta)l(d+r) \\ -(R_s f_s + R_m f_m)e - S(\beta)\} + \mu\{(d+r)l - [-q_s(1-l) + q_m(1-l)(1-\theta)]R_s e \\ -[-(1-q_s)(1-l) + (1-q_m)(1-l)(1-\theta)]R_m e - S'(\beta)\}. \end{aligned}$$

Then the optimal solution satisfies the following first-order conditions:

$$\frac{\partial F}{\partial R_s} = f_s e - \lambda f_s e - \mu[-q_s(1-l) + q_m(1-l)(1-\theta)]e = 0, \tag{30}$$

$$\frac{\partial F}{\partial R_m} = f_m e - \lambda f_m e - \mu[-(1-q_s)(1-l) + (1-q_m)(1-l)(1-\theta)]e = 0, \tag{31}$$

$$\frac{\partial F}{\partial d} = (1-\beta)l - \lambda(1-\beta)l + \mu l = 0. \tag{32}$$

Solving (30)–(32), we obtain $\lambda = 1$ and $\mu = 0$. Substituting $\lambda = 1$ and $\mu = 0$ into the Lagrange function F and differentiating the result with respect to β, θ, and l once, we have

$$\frac{\partial F}{\partial \beta} = lr + \theta(1-l)e - S'(\beta) = 0, \tag{33}$$

$$\frac{\partial F}{\partial \theta} = [1 - (1-\beta)(1-l)]e - M'(\theta) = 0, \tag{34}$$

$$\frac{\partial F}{\partial l} = -(1-\beta)r + \theta(1-\beta)e - I'(l) = 0. \tag{35}$$

It is easy to verify that the centralised optimal quality levels (β^*, θ^*, l^*) are a solution to (33)–(35) by comparing them with (4)–(6). Then substituting (β^*, θ^*, l^*) into the IC and IR constraints, we get

$$\omega - (1-\beta^*)l^*(d+r) - (R_s f_s + R_m f_m)e - S(\beta^*) = 0, \tag{36}$$

$$\begin{aligned} l(d+r) - [-q_s(1-l^*) + q_m(1-l^*)(1-\theta^*)]R_s e - [-(1-q_s)(1-l^*) \\ +(1-q_m)(1-l^*)(1-\theta^*)]R_m e - S'(\beta^*) = 0. \end{aligned} \tag{37}$$

Solving (36) and (37), we get R_s^{*S} and R_m^{*S}. Furthermore, d should be properly chosen to ensure that R_s^{*S} and $R_m^{*S} \in [0,1]$. Proposition 2 is established.

Proof of Proposition 3

Note that the IR constraint must be binding; otherwise, the manufacturer can improve its profit by increasing the price rebate d or decreasing the wholesale price ω. In addition, π_s^P is concave in β, while π_m^P is concave in θ. The double moral hazard programme can be rewritten as

$$\begin{array}{ll} \max\limits_{\theta,l,\omega,d,R_s,R_m} & \pi_m^P(\beta,\theta,l,d,\omega,R_s,R_m) \\ \text{s.t.}\quad (\text{IR}) & \pi_s^P(\beta,\theta,l,d,\omega,R_s,R_m)=0, \\ \quad (\text{IC}) & \partial\pi_s^P(\beta,\theta,l,d,\omega,R_s,R_m)/\partial\beta=0, \\ \quad (\text{IC}') & \partial\pi_m^P(\beta,\theta,l,d,\omega,R_s,R_m)/\partial\theta=0, \\ \quad 0\le & \beta,\theta,l,R_s,R_m\le 1, d\ge 0. \end{array} \tag{38}$$

The above-constrained optimisation problem can be solved by the method of Lagrange multipliers. The Lagrange function is

$$\begin{aligned} G = {} & p+(1-\beta)ld-\omega-\{(1-R_s)f_s+(1-R_m)f_m\}e-M(\theta)-I(l) \\ & +\lambda'\{\omega-(1-\beta)l(d+r)-(R_sf_s+R_mf_m)e-S(\beta)\}+\mu'\{(d+r)l-[-q_s(1-l) \\ & +q_m(1-l)(1-\theta)]R_se-[-(1-q_s)(1-l)+(1-q_m)(1-l)(1-\theta)]R_me-S'(\beta)\} \\ & +\gamma'\{(1-R_s)[1-(1-\beta)(1-l)]q_me+(1-R_m)[1-(1-\beta)(1-l)](1-q_m)e-M'(\theta).\} \end{aligned}$$

The optimal solution $(\beta^{*D},\theta^{*D},l^{*D},d^{*D},R_s^{*D},R_m^{*D})$ must satisfy the first-order conditions. First differentiating G with respect to d, R_m, and R_s once, we have

$$\frac{\partial G}{\partial R_s}=f_se-\lambda' f_se-\mu'[-q_s(1-l)+q_m(1-l)(1-\theta)]e-\gamma' q_m[1-(1-\beta)(1-l)], \tag{39}$$

$$\begin{aligned} \frac{\partial G}{\partial R_m}= {} & f_me-\lambda' f_me-\mu'[-(1-q_s)(1-l)+(1-q_m)(1-l)(1-\theta)]e \\ & -\gamma'(1-q_m)[1-(1-\beta)(1-l)]e=0, \end{aligned} \tag{40}$$

$$\frac{\partial G}{\partial d}=(1-\beta)l-\lambda'(1-\beta)l+\mu' l=0. \tag{41}$$

Solving (39)–(41), we get $\lambda'=1,\mu'=0$, and $r'=0$. Then substituting $\lambda'=1,\mu'=0$, and $r'=0$ into the Lagrange function G, and differentiating the result with respect to β, θ, and l once, we get $\beta^{*D}=\beta^*$, $\theta^{*D}=\theta^*$ and $l^{*D}=l^*$.

R_s, R_m and d should satisfy the IR, IC and IC' constraints in (38). Rewriting the IC' constraint, we get

$$R_sq_m+R_m(1-q_m)=1-\frac{M'(\theta^*)}{[1-(1-\beta^*)(1-l^*)]e}=0, \tag{42}$$

where the second equation is due to (5). Considering $R_s \ge R_m$, we obtain the solution to (42), i.e. $q_m=0$ and $R_m=0$. Substituting $q_m=0$ and $R_m=0$ into IR and IC, we get

$$\omega^{*D}=S(\beta^*)+(1-\beta^*)S'(\beta^*), \tag{43}$$

$$R_s^{*D}=\frac{\theta^*(1-l^*)e-d^{*D}l^*}{q_s(1-l^*)e}, \tag{44}$$

where the price rebate d should be appropriately chosen to ensure that $0\le R_s^{*D}\le 1$. Therefore, d^{*D} should satisfy $\frac{(1-l^*)(\theta^*e-q_se)}{l^*}\le d^{*D}\le\frac{\theta^*(1-l^*)e}{l^*}$.