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Production planning and pricing policy in a make-to-stock system with uncertain demand subject to machine breakdowns

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ABSTRACT

We consider a make-to-stock system served by an unreliable machine that produces one type of product, which is sold to customers at one of two possible prices depending on the inventory level at the time when a customer arrives (i.e., the decision point). The system manager must determine the production level and selling price at each decision point. We first show that the optimal production and pricing policy is a threshold control, which is characterized by three threshold parameters under both the long-run discounted profit and long-run average profit criteria. We then establish the structural relationships among the three threshold parameters that production is off when inventory is above the threshold, and that the optimal selling price should be low when inventory is above the threshold under the scenario where the machine is down or up. Finally we provide some numerical examples to illustrate the analytical results and gain additional insights.

Introduction

Manufacturing systems normally experience random failures and recoveries during their operation. This issue of machine unreliability and its impact present great challenges to manufacturing organizations and operations. These challenges have become increasingly significant in the context of just-in-time manufacturing (JIT-M), which is widely applied in the manufacturing industry. JIT-M is susceptible to disruptions due to machine (or part) failures, labor strikes, and unavailability of raw materials, among other causes. Such disruptions will reduce the system's effective capacity and may result in high operational costs. Examples of the negative impact of process disruptions on firms' operations can be found in the automobile and semiconductor manufacturing industries (see, e.g., Stern, 1994; Hamilton, 1993; Parlar & Berkin., 1991). On the other hand, facing an increasingly complex marketing environment, how to match supply with demand has become a crucial issue for firms to address. In the real world, adjusting prices is an effective means to mitigate the demand–supply mismatch. Making joint production and pricing decisions is quite common in some industries such as electronic product manufacturing. For example, production expansion plans in the liquid crystal display (LCD) and LED panel manufacturing industries typically extend to over two years. On the one hand, the expansion plan needs to acquire reliable capacity to cater for the booming market trend. On the other hand, there is a risk of over-supply. While it is difficult to solve the over-supply problem in the short term, firms can use pricing to match demand with desirable inventory levels.

Motivated by industrial observations, we investigate the impact of random disruptions in a bottleneck production facility in this paper. Besides supply management, the system manager also adopts dynamic pricing to adjust customer demand. Formulating the problem as a Markovian decision process, we consider the joint management of production and the demand process in the setting of a manufacturing system served by a failure-prone machine. The production process is random with a changeable mean production rate and the machine may break down at any time. The demand process is stochastic with a changeable mean demand rate, which depends on the product selling price being high or low. The problem is to maximize either the long-run discounted profit or the long-run average profit where the production rate and the unit selling price are random variables. At each decision point, we first decide how to dynamically adjust the production rate when the machine is up, then determine to apply the high or low price according to the current production rate and the status of the machine.

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We review two streams of research pertinent to the two research questions under study as follows: Failure-prone manufacturing systems have been extensively studied in the last two decades (see, e.g., the recent works of Cai, Wu, & Zhou, 2009, 2011). Buzacott and Shanthikumar (1993) classified the related literature into two groups. The first group is based on the "fluid flow model", where production is modeled as a continuous process and the analysis often assumes constant demand and processing times. It has been shown that the optimal policy under such a model is a hedging point control if unmet demands are completely backlogged (see, e.g., Akella & Kumar, 1986; Bielecki & Kumar, 1988). These results were extended to systems without backlogging by Hu (1995) and Mohebbi (2006) or bounded backlogging by Martinelli and Valigi (2004). A comprehensive list of works in this group of studies can be found in Gershwin (1994) and Sethi and Sethi (1990). The second group is based on the "discrete part manufacturing system", where parts are produced in discrete mode. This group takes into account the extra randomness in the production times and demand arrivals. With the assumption that unmet demands are fully backlogged, Song and Sun (1999), Feng and Yan (2000), Feng and Xiao (2002), Song (2006) and Chiu (2008) established the optimality of a hedging point policy in failure-prone manufacturing systems and provided the explicit forms of the discounted cost and the average cost under a threshold policy. Focusing on production decision making, these studies seldom discuss the pricing decision issue.

We consider in this paper both the production issue and dynamic pricing strategies in the setting of a failure-prone manufacturing system. In view of the fact that dynamic pricing strategies are widely adopted in various industries, Elmaghraby and Keskinocak (2003) characterized the market environments in which dynamic pricing is adopted in practice. Chen, Feng, and Ou (2006) considered the joint management of production and the demand process for a random production system with inventory replenishments. Feng, Ou, and Pang (2008) studied the optimal control of an assemble-to-order system with multiple make-to-stock components and a variable pricing policy. The interested readers may refer to Chen, Chen, and Pang (2011), Keblis and Feng (2012) and Pang, Chen, and Feng (2012) for more discussions of the dynamic inventory-pricing control problem.

There are studies on the impact of production time variability on the optimal inventory control policy with stochastic demand based on numerical analysis (see Sanajian, 2009). Another stream of research considers the joint management of capacity and inventory with stochastic demand in the make-to-order (MTO) system (Mayorga, Ahn, & Shanthikumar (2006), Gayon, de Vricourt, & Karaesmen (2009) and Mayorga & Ahn (2011)). The customer classes are defined by the backorder cost, rather than being differentiated by the pricing policy as in our work. Li and Womer (2012) address issues similar to ours in the MTO system.

Our setting differs from these studies in that our production system is unreliable under the make-to-stock (MTS) scenario, i.e., the machine may break down at any time. In reality, it is reasonable for a manufacturer to make joint production and pricing decisions to cope with the difficulties caused by limited resource capacity and various uncertainties such as random demand arrivals, unreliable machines, and stochastic processing times.

Our study was inspired by Ha (1997, 1997), which apply the Markov decision process approach presented in Porteus (1982). Focusing on cost parameters, the former two papers study a scheduling problem with two products and a stock rationing problem with several demand classes, respectively. However, we consider a different model setting from that in Ha (1997, 1997), whereby we focus on the pricing policy in a failure-prone manufacturing system.

Combining production control and price switching control into an integrated policy in this paper, we show that the production and pricing policy is a threshold control, which can be characterized by three threshold parameters. The first one is the production policy, which is the base-stock type, so production is off when inventory is above a certain level. The other two thresholds are the optimal control of the selling price under the scenarios where the machine is down and up, respectively. The decision maker will sell the product at a high price when inventory is low, and vice versa. The result that the optimal control policy is a threshold control characterized by three parameters holds true for both the discounted profit and long-run average profit cases. We establish the structural relationships among the three threshold parameters. Our findings imply that the manufacturer should not presume an extremely high repair rate because the related cost is high and the effect of dynamic pricing becomes weak as a result.

This paper is organized as follows: We formulate the problem as an event-driven Markov decision process in section 'The model'. We show the optimality of a threshold control for the discounted profit and the average profit cases in section 'Structure of the optimal policy'. We present a numerical example to illustrate the analytical results and gain additional insights in section 'Numerical analysis'. section 'Conclusions' concludes the paper with a summary of the results and a discussion of possible extensions of this study.

The model

There is a manufacturer that produces a single product to stock using an unreliable machine. The product is sold to customers at one of two possible prices depending on the inventory level at the time when a customer arrives. It follows that the underlying make-to-order system is subject to random machine breakdowns. When the machine is up, the system produces the product at a unit production cost c and its unit production time is exponentially distributed with a rate μ, where μ is adjustable within the range $\mu \in [0, r]$ and r is the maximum production rate. When the machine is down, the system produces nothing and the machine is sent to repair immediately. The production interrupted by machine breakdowns is resumed once the machine is repaired. Due to the memoryless property of the exponential processing time, the remaining processing time is stochastically equivalent to initiating production from scratch. Following Bielecki and Kumar (1988) and Posner and Berg (1989), we assume that the machine operating time until breakdown (i.e., machine up time) is exponentially distributed with an average failure rate q_0 and the machine repair time is exponentially distributed with an average repair rate q_1, where the average failure rate of the machine is the inverse of the mean time to failure (MTTF) and the average repair rate of the machine is the inverse of the mean time to repair (MTTR), i.e., $q_0 = \frac{1}{MTTF}$ and $q_1 = \frac{1}{MTTR}$.

Following Chen et al. (2006) and Feng et al. (2008), we model the product demand as a non-homogeneous Poisson process with a mean rate dependent on the selling price at the time when a customer arrives (i.e., the decision point). For the price setting problem, it is not always feasible to change prices every day or period because (1) it is difficult and expensive to update demand forecasting and pricing decisions and (2) frequently changed prices are not acceptable to downstream members of the supply chain. Although flexible pricing may maximize profit for the manufacturer, a simple pricing policy is often used in the real world whereby a high price is imposed in the peak demand season while a low price is adopted to spur demand. According to the latest *2013 Global Sapphire Substrate Market Report*, LED chip manufacturers apply a relatively stable pricing policy in recent years (see Fig. 1). From

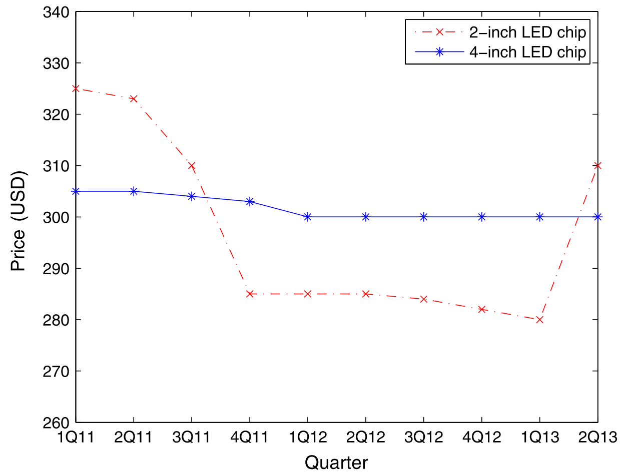

Fig. 1. Price trend in the LED industry (*Source:* LED inside).

the monthly sales data of Samsung and BenQ, two global manufacturers in the LED industry, we see that they adopt a high/low pricing policy for a certain duration, the length of which is more than a week. In the peak demand period, their selling prices are \$270 and \$210 per unit, respectively, while they reduce the prices to \$200 and \$110 when the demand is low. Based on this industrial observation, we assume that there are two possible price levels to sell the product, namely high price p_1 and low price p_2 with the corresponding demand rates λ_1 and λ_2, respectively. To make the low price economically reasonable, we assume that $p_1 > p_2 > c$ and $0 \leqslant \lambda_1 < \lambda_2$. We focus on the scenario where $p > c$ because the manufacturer will produce nothing in situations where any nonzero production is unprofitable.

We also assume that any unmet demand can be backordered. If the finished goods inventory is positive, the demand is satisfied instantaneously and a revenue of p_1 or p_2 is realized (dependent on the product selling price charged to the customer at the customer arrival time). Otherwise, the demand is added to the order waiting queue, which is served in the first-come-first-served order. We assume that the average capacity is no less than the high demand rate, i.e., $\frac{q_0}{q_0+q_1} r \geqslant \lambda_2$, to ensure system stability as all the demand can be fulfilled in the long run. To make the price switch as a possible policy, we further denote $\xi_{12} = \frac{\lambda_2 p_2 - \lambda_1 p_1}{\lambda_2 - \lambda_1}$, which can be viewed as the marginal revenue gain (loss) when the product selling price is switched from high to low (low to high).

We use $(x(t), i(t))$ to denote the system states, where $x(t) \in I$ is the inventory level of the finished goods at time t and $i(t) \in \{0,1\}$ is the machine state at t, i.e.,

$$i(t) = \begin{cases} 1, & \text{if the machine is up at time } t, \\ 0, & \text{if the machine is failure at time } t. \end{cases}$$

The finished products held in inventory incur a holding cost of h per unit product per unit time and the backordering cost for the demand unmet at arrival is b per waiting demand per unit time. Denote the inventory related cost function as $g(x) = hx^+ + bx^-$, where $x^+ = \max(0,x)$ and $x^- = \max(0,-x)$.

At time t, there are two types of decision to make: the first is to choose a production rate $\mu(t)$ in $[0,r]$ when $i(t)=1$, and $\mu(t)=0$ when $i(t)=0$; the second is to choose the product selling price $p(t)$ at p_1 or at p_2. Denote the policy as $u = \{\mu(t), p(t) : t > 0\}$, and call it *nonanticipatory* if, at any $t > 0$, $\mu(t)$ and $p(t)$ depend only on the information prior to t. Let U be the collection of all such nonanticipatory control policies. Given a control policy $u \in U$, we record the realized total demand sold at price p_k up to time $t > 0$ as $N^u_k(t), k = 1,2$; the realized total production output as $P^u(t)$; and the realized finished goods inventory level process as $x^u(t)$. Then given an initial inventory level x, an initial machine state i, and a control policy u, the finite horizon expected discounted profit, the infinite horizon expected discounted profit, and the long-run average profit can be represented, respectively, as

$$J^u_{T,\gamma}(x,i) = E\left\{\int_0^T e^{-\gamma t}\left[\sum_{k=1}^{2} p_k dN^u_k(t) - cdP^u(t) - g(x^u(t))\right] dt \,|\, x(0)=x, i(0)=i\right\}, \tag{1}$$

$$J^u_d(x,i) = \lim_{T\to\infty} J^u_{T,\gamma}(x,i), \tag{2}$$

$$J^u_a(x,i) = \lim_{T\to\infty} \frac{J^u_{T,0}(x,i)}{T}, \tag{3}$$

where $0 < \gamma < 1$ is a discounted factor, and we assume that $c < b/\gamma$. The problem is to find the optimal control $u(t) \in U$ to maximize the total expected discounted profit $J^u_d(x,i)$ or the long-run average profit $J^u_a(x,i)$.

A policy $u^*_d \in U$ is said to be optimal under the long-run discounted criterion if it maximizes the expected total discounted profit (2), i.e., it solves the following optimization problem:

$$J^{u^*_d}(x,i) = \max_{u\in U} J^u_d(x,i), i = 0,1. \tag{4}$$

Similarly, a policy $u^*_a \in U$ is said to be optimal under the long-run average criterion if it maximizes the expected long-run average profit (3), i.e., it solves the following optimization problem:

$$J^{u^*_a}(x,i) = \max_{u\in U} J^u_a(x,i), i = 0,1. \tag{5}$$

Structure of the optimal policy

Optimal control for the long-run discounted profit

In this section we aim at finding the optimal control policy for the long-run discounted profit case. We re-scale the time unit so that $\gamma + \lambda_1 + \lambda_2 + q_0 + q_1 + r = 1$. To simplify notation, we let $J(x,i) = J^{u^*_d}(x,i)$ denote the maximum total discounted profit function of the optimal control policy. Then according to the theory of Markov decision processes, $J^{u^*_a}(x,i)$ is the solution to the following Bellman–Hamilton–Jacobi equations:

$$J(x,1) = -g(x) + q_0 J(x,0) + q_1 J(x,1) + HJ(x,1) + rAJ(x,1), \tag{6}$$
$$J(x,0) = -g(x) + q_0 J(x,0) + q_1 J(x,1) + rJ(x,1) + HJ(x,0), \tag{7}$$

where

$$\begin{aligned} Av(x,1) &= \max(v(x,1), v(x+1,1) - c) \\ &= v(x,1) + [v(x+1,1) - v(x,1) - c]^+, \end{aligned}$$

$$\begin{aligned} Hv(x,1) &= \max(\lambda_1 p_1 + \lambda_1 v(x-1,1) + \lambda_2 v(x,1), \lambda_2 p_2 \\ &\quad + \lambda_2 v(x-1,1) + \lambda_1 v(x,1)) \\ &= \lambda_1 [p_1 + v(x-1,1)] + \lambda_2 v(x,1) + (\lambda_2 - \lambda_1) \\ &\quad [\xi_{12} - (v(x,1) - v(x-1,1))]^+, \\ Hv(x,0) &= \max(\lambda_1 p_1 + \lambda_1 v(x-1,0) + \lambda_2 v(x,0), \lambda_2 p_2 \\ &\quad + \lambda_2 v(x-1,0) + \lambda_1 v(x,0)) \\ &= \lambda_1 [p_1 + v(x-1,0)] + \lambda_2 v(x,0) + (\lambda_2 - \lambda_1) \\ &\quad [\xi_{12} - (v(x,0) - v(x-1,0))]^+. \end{aligned}$$

Specifically, A corresponds to the production control and H corresponds to the price switching control. For example, whenever $J(x+1,1) - c > J(x,1)$ when the current inventory is x, it is optimal to produce the product. We first derive the properties of the solution to (6) and (7), and then identify the characteristic structures of the corresponding optimal control policies.

To simplify notation in this section, we define for any function $v(x,i) \in Z \times \{0,1\}$ and the operator T,

$$Tv(x,i) = \begin{cases} -g(x)+q_0v(x,0)+q_1v(x,1)+Hv(x,1)+rAv(x,1), & i=1, \\ -g(x)+q_0v(x,0)+q_1v(x,1)+rv(x,1)+Hv(x,0), & i=0. \end{cases}$$

Let V be the set of functions $v(x,i) \in Z \times \{0,1\}$ with the following properties:

C1: Concavity: $Dv(x+1,i) - Dv(x,i) \leqslant 0$, for both $i = 0,1$.
C2: $v(x,1) \geqslant v(x,0)$, for any x; $Dv(x,1) \leqslant Dv(x,0)$.
C3: $Dv(x+1,1) > c$, if $x < 0$.
C4: Linear limits:

$$\lim_{x\to+\infty} Dv(x,i) = -\frac{h}{\gamma}, i = 0,1, \qquad \lim_{x\to-\infty} Dv(x,i) = \frac{b}{\gamma}, \quad i = 0,1,$$

where

$$Dv(x,i) = v(x,i) - v(x-1,i), \qquad D^2v(x,i) = Dv(x,i) - Dv(x-1,i).$$

$C3$ implies that the facility does not stop production if it causes any backlog.

The following lemma shows that the operator T preserves the structure of the functions in V.

Lemma 1. *If $v(x,i) \in V$, then $Tv(x,i) \in V$, for $i = 0,1$.*

Proof. See the Appendix. □

Based on Lemma 1, we provide a theorem on the maximum profit that it belongs to V and, more importantly, it satisfies the four properties of V. We then obtain the optimal production control and pricing policies.

Theorem 1. *The maximum profit function of $J(x,i) \in V, i = 0,1$.*

Proof. The maximum profit function $J(x,i)$ exists and it satisfies conditions $C1$—$C4$ following the analysis of Ha (1997, 1997), which is based on the result on the Markov decision process as summarized in Porteus (1982). In particular, according to Theorem 5.1 in this paper, $J(x,i) = \lim_{n\to\infty} T^n v(x,i)$ for a function $v(x,i)$ that is well-defined and it is the unique solution to the Bellman–Hamilton–Jacobi Eqs. (6) and (7) for optimality. □

Theorem 2. *The optimal production control and pricing policy $u^* = (\mu^*(x,i), p^*(x,i))$ is a threshold-type control, characterized by three threshold parameters d^*, R_0^*, and R_1^*, i.e.,*

$$\mu^*(x,1) = \begin{cases} r, & x < d^* \\ 0, & x \geqslant d^* \end{cases} \qquad \mu^*(x,0) = 0, \tag{8}$$

$$p^*(x,1) = \begin{cases} p_1, & x \leqslant R_1^* \\ p_2, & x > R_1^* \end{cases} \qquad p^*(x,0) = \begin{cases} p_1, & x \leqslant R_0^*, \\ p_2, & x > R_0^*, \end{cases} \tag{9}$$

where $d^* = \max\{s | J(s,1) - J(s-1,1) > c\}$, $R_1^* = \max\{s | J(s,1) - J(s-1,1) > \xi_{12}\}$, and $R_0^* = \max\{s | J(s,0) - J(s-1,0) > \xi_{12}\}$. In addition, we have $d^* \geqslant 0$ and $R_1^* \leqslant R_0^*$.

Proof. The threshold control structure of the optimal policy is the direct result of Lemma 1. Moreover, property $C3$ yields $d^* \geqslant 0$ and property $C2$ yields $R_1^* \leqslant R_0^*$. □

From Theorem 2, (8) implies that the optimal control of the long-run discounted case is a base-stock policy, i.e., production is off when inventory is above a certain base-stock level. In addition, (9) implies that the optimal control of the product selling price is of the threshold type, i.e., high (low) price is chosen when inventory is low (high). Moreover, the theorem also implies that the pricing threshold for the machine breakdown case is higher than that for the case where the machine is up.

Optimal control for the long-run average profit

In the previous section we showed that under the discounted profit criterion, the optimal control policy for the system can be specified by a base-stock level and a price switch threshold point. In this section we aim at establishing the structural characteristics of the optimal policy for the long-run average profit case. As mentioned before, the average capacity is no less than the high demand arrival rate, i.e., $\frac{rq_0}{q_0+q_1} \geqslant \lambda_2$, so the system is stable. It follows that the induced Markov chain is positive recurrent. Therefore the average cost optimality equations are obtained as follows:

$$\omega(x,1) + J^* = -g(x) + q_0\omega(x,0) + q_1\omega(x,1) + H\omega(x,1) + rA\omega(x,1), \tag{10}$$

$$\omega(x,0) + J^* = -g(x) + q_0\omega(x,0) + q_1\omega(x,1) + r\omega(x,1) + H\omega(x,0), \tag{11}$$

where J^* is the long-run optimal average profit and $\omega(x,i)$ is a finite function.

Lemma 2. *With $\omega(x,i)$ defined above, we have*

$a1$. $D\omega(x,i) \leqslant D\omega(x-1,i)$, for both $i = 0,1$;
$a2$. $\omega(x,1) \geqslant \omega(x,0)$, for all x; $D\omega(x,1) \leqslant D\omega(x,0)$;
$a3$. $D\omega(x+1,1) \geqslant c$, if $x < 0$;
$a4$. Linear limits:

$$\lim_{x\to+\infty} D\omega(x,i) = -\infty, i = 0,1,$$
$$\lim_{x\to-\infty} D\omega(x,i) = \infty, i = 0,1.$$

Proof. We take advantage of the fact that the average profit can often be obtained as the limit of the discounted profit problem. Noting that $\omega(x,i) = \lim_{\gamma\to 0}(J(x,i) - J(d^*,i)$ by Bielecki and Kumar (1988) and recalling the results of $J(x,i)$, we obtain the results. □

Lemma 2 provides the properties of $\omega(x,i)$, which has a similar structure as that of $v(x,i)$ in the discounted profit problem. Similarly, we obtain a theorem on the optimal production control and pricing policy as follows:

Theorem 3. *The optimal production control and pricing policy $u^* = (\mu^*(x,i), p^*(x,i))$ for the long-run average cost is a threshold-type control characterized by three threshold parameters d^*, R_0^*, and R_1^*, i.e.,*

$$\mu^*(x,1) = \begin{cases} r, & x < d^* \\ 0, & x \geqslant d^* \end{cases} \qquad \mu^*(x,0) = 0, \tag{12}$$

$$p^*(x,1) = \begin{cases} p_1, & x \leqslant R_1^* \\ p_2, & x > R_1^* \end{cases} \qquad p^*(x,0) = \begin{cases} p_1, & x \leqslant R_0^*, \\ p_2, & x > R_0^*, \end{cases} \tag{13}$$

where $d^* = \max\{s | \omega(s,1) - \omega(s-1,1) > c\}$, $R_1^* = \max\{s | \omega(s,1) - \omega(s-1,1) > \xi_{12}\}$, and $R_0^* = \max\{s | \omega(s,0) - \omega(s-1,0) > \xi_{12}\}$. In addition, we have $d^* \geqslant 0$ and $R_1^* \leqslant R_0^*$.

From Theorem 3, (12) implies that the optimal control for the long-run average case is also of the base-stock type, i.e., production is off when inventory is high, above a certain base-stock level. In addition, (13) implies that the optimal control of the product selling price is of the threshold type, i.e., a high (low) price is chosen when inventory is low (high). It also implies that the threshold point chosen when the machine is down is higher than that when the machine is up.

Numerical analysis

In this section we present some numerical examples to illustrate the analytical results and gain additional insights. We also show how the optimal control policy varies with changes in the system parameters. In addition, we compare the performance of our model with that using the optimal pricing policy in each period to further show the joint effects of the pricing and production decisions.

The optimal profit function and the optimal control policy under the long-run discounted profit criterion or the long-run average cost criterion can be calculated by the value iteration algorithm. The reader is referred to Kimemia and Gershwin (1983) for details on the value iteration algorithm. In order to apply the value iteration algorithm, the infinite countable state space is truncated to $[-S_1, S_2]$, where both S_1 and S_2 are sufficiently large to ensure that the optimal profit function is not sensitive to the truncated state space.

We present a numerical example of a simple failure-prone manufacturing system. By using the value iteration algorithm, we obtain the optimal base-stock level and optimal threshold levels under the long-run discounted profit criterion. We consider the case with parameters: $\lambda_1 = 0.4, \lambda_2 = 0.6, p_1 = 60, p_2 = 50, q_0 = 0.2, q_1 = 0.1, h = 1.2, b = 20, r = 1,$ and $c = 10$. Fig. 2 shows how the system parameters affect the optimal control policy.

This figure indicates that the base-stock level and the threshold levels have the relationship as given by Theorem 1, i.e., $R_1 \leqslant R_0$ and $d \geqslant 0$. Fig. 2(1) and (2) shows how the inventory-related costs affect the optimal control policy. We see that it is optimal to lower the base-stock level and threshold levels when the holding cost h is high, while raising the levels when the backlog cost is high. In particular, when the holding cost is larger than 6, R_1 drops to zero. This result implies that the firm has a strong motivation to clear the stock due to the high holding cost, so all the products are sold at the low price p_2. In addition, we also observe that the optimal stock level d is more sensitive towards the holding cost h than the penalty cost b. Specifically, the value of d decreases significantly as h varies from 0 to 5. This observation provides the managerial insight that the unmet demand is backlogged in an order waiting queue, which reduces the uncertainty to a certain extent, while the firm bears more risk to keep high inventory facing stochastic demand with increasing holding cost.

Fig. 2(3) illustrates that the base-stock level and price threshold levels increase with the failure rate. The base-stock level and threshold levels increase to cope with the increasing failure rate, which means machine breakdown plays a key role and affects the system profit.

Fig. 2(4) shows that the base-stock level and price threshold levels decrease with the repair rate, and the difference between the threshold levels trends to become smaller and approaches zero

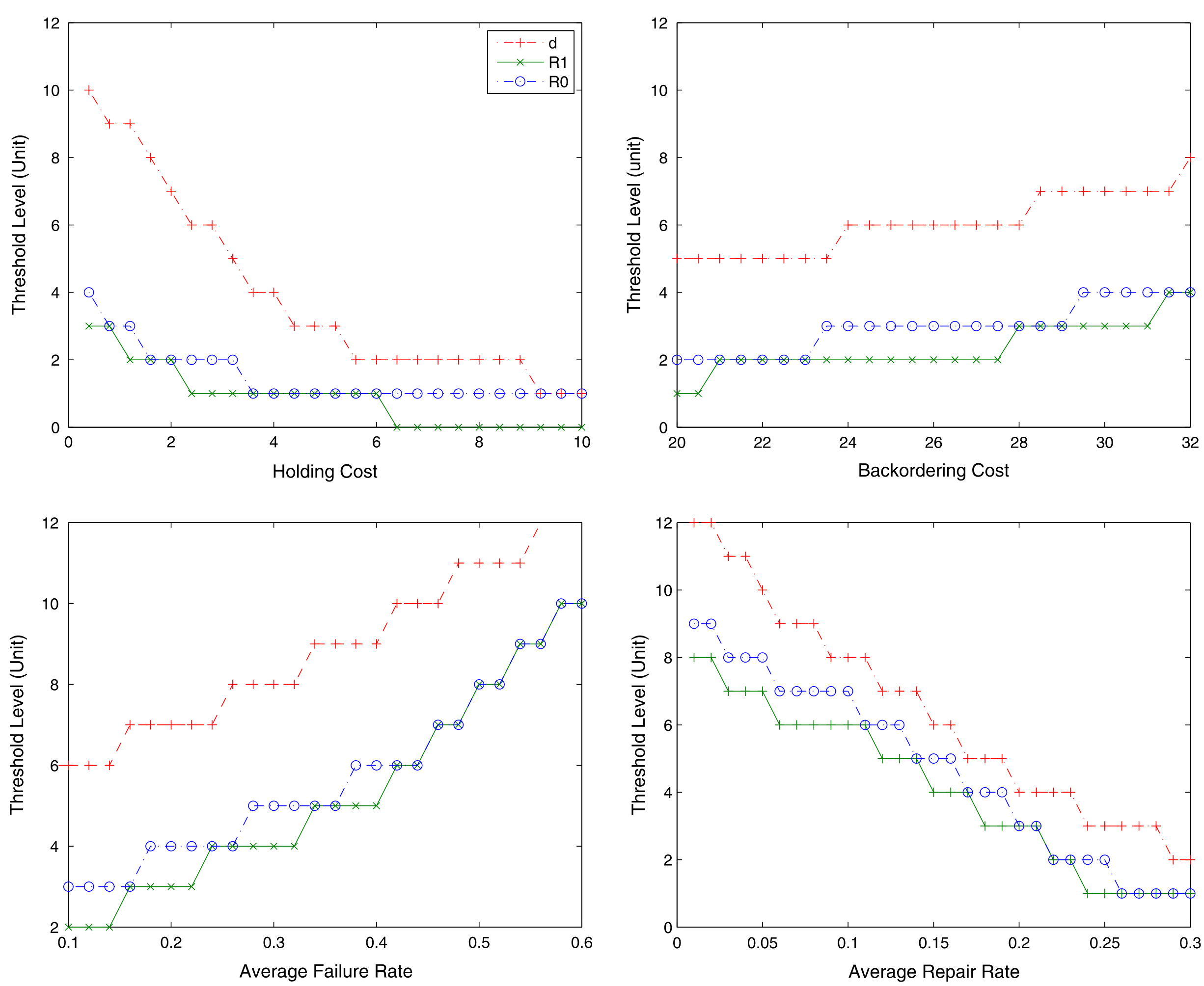

Fig. 2. Impact of (1) holding cost (h), (2) backlogging cost (b), (3) failure Rate (q_0), and (4) repair rate (q_1) on the threshold levels (d, R_1, R_0).

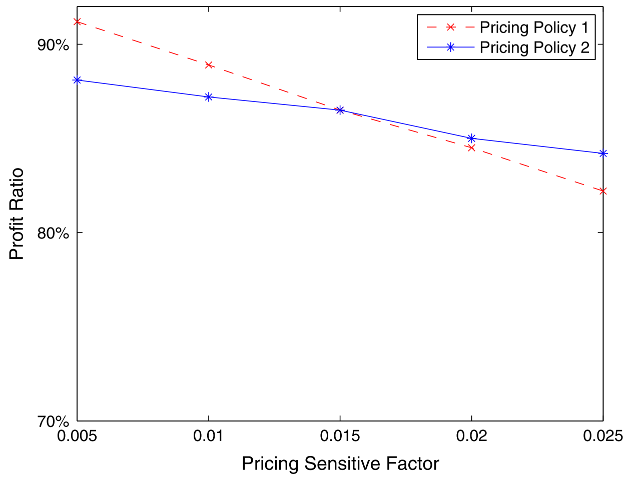

Fig. 3. Profit comparison of the high/low policy.

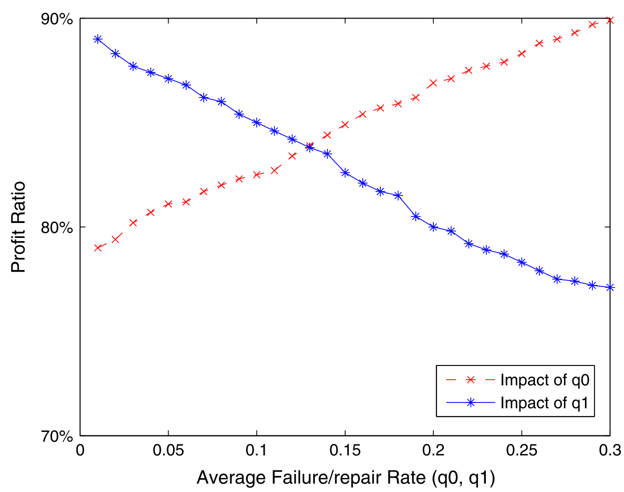

Fig. 4. Impact of average failure rate (q_0) and average repair rate (q_1).

as the repair rate grows. This result can be explained as follows: As the repair rate increases, the effect caused by machine failures becomes small, and there is no need to reserve so much inventory to cope with replenishment uncertainty. When the repair rate reaches a certain level, the failed machine can be repaired in a very short time or short enough compared with the demand inter-arrival times and processing times, so the failure aspect can be ignored, and the threshold levels for different machine states trend to be equal. On the other hand, as the repair rate q_1 increases and the optimal base-stock level decreases, the thresholds R_0 and R_1 drop to 1 as a result. This result implies that since machine failure becomes less important, the manufacturer meets stochastic demand with stable production. Accordingly, the firm attempts to charge the lower price p_2 to avoid over-stocking. However, from the managerial perspective, choosing a low price to increase sales does not always benefit the manufacturer because the market cannot be properly charged. In addition, based on the common sense that cost increases at a high repair rate q_1, it is not advisable to keep an extremely high repair rate.

Next, based on the basic data setting, we further consider the scenario where the optimal pricing policy is used as the benchmark. Assume that the demand rate is $\lambda = 1 - \beta p$, where β reflects the pricing sensitivity. We analyze the effects of the two-price policy, machine failure rate, and repair rate on the optimal decisions. First, we show the effects of the two pricing policies on the optimal producing decisions in Figs. 3 and 4.

In our model we assume that the manufacturer adopts the high/low pricing policy to adjust the demand. We now analyze how to decide the gap between the high and low prices. Assume that the price gap is small (i.e., 5) under policy 1, while it is large (i.e., 20) under policy 2. We compare the profits under the two scenarios with that under the optimal pricing policy. In Fig. 3, the profit ratio denotes the profit obtained under the high/low policy to the benchmark. This implies that the profit in the high/low case is quite close to the optimized one, when the value of the price-sensitive factor β is low. Moreover, it shows that when β is low, the manufacturer should choose a smaller gap between the two prices, while enlarging the gap as β increases.

We illustrate the effects of q_0 and q_1 on the profit in Fig. 4. Although the profit obtained under the high/low policy is close to the optimal one, the gap between them decreases in q_0 while increases in q_1. As the failure rate increases, Fig. 2(3) shows that the base-stock level increases. Hence, an adequate inventory level weakens the role of the flexible pricing policy and the profits in the two cases are close to each other. On the other hand, since the base-stock level decreases in q_1, the role of the optimal price becomes more significant, which causes an increase in the profit gap. As shown in the figure, the profit ratio is low when q_1 is large enough. However, Fig. 2(4) implies that an extremely high repair rate is not necessary as the high/low policy in our model is still acceptable.

Conclusions

This paper considers a production control problem with a failure-prone machine. The control decisions include determining the production rate when the machine is up and whether to sell the product at a high price or a low price. We propose a discrete Markovian production model and derive the control policies under both the long-run discounted profit and long-run average profit criteria. For both cases, we prove that the optimal policies are of the threshold control type and can be characterized by three threshold parameters. We establish the structural relationships among the three threshold parameters. First, the optimal production policy is the base-stock policy, i.e., production is off when inventory is above a certain base-stock level. Second, the optimal control of the product selling price is also of the threshold type, i.e., a high (low) price is chosen when inventory is low (high). Moreover, the price-changing threshold parameter when the machine is down becomes higher than that when the machine is up. We also observe that with the backlogging assumption, the manufacturer facing stochastic demand bears more risk in keeping high inventory, so the optimal stock level is more sensitive towards the holding cost than the penalty cost. Moreover, the numerical analysis also suggests that it is not advisable to keep a high repair rate because the operational cost will increase and the market cannot be properly charged for keeping high inventory.

Based on the threshold-type optimal control policy, we find that it is advantageous to leave the selling price unset and optimize it in the failure-prone manufacturing system setting. For future research, one may further analyze the problem using a more general pricing strategy. In addition, one may also extend the model to the lost sales case, and compare the performance after deriving the explicit form of the stationary distribution under the threshold policy. This work provides the basic perspective for the production and pricing policy with machine failure considerations. Furthermore, we assume that the random production process follows the exponential distribution. Studying the problem with general production and breakdown distributions is an important topic for further research. Topics along this line include extending the model to the cases where there are multiple machines or products.

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Proof of Lemma 1

Verification of C1:

For A, we have

$$\begin{aligned}
D^2Av(x,1) &= D^2\{v(x,1)+[v(x+1,1)-v(x,1)-c]^+\}\\
&= D^2v(x,1)+[v(x+1,1)-v(x,1)-c]^+\\
&\quad -2[v(x,1)-v(x-1,1)-c]^+\\
&\quad +[v(x-1,1)-v(x-2,1)-c]^+\\
&\leqslant D^2v(x,1)-[v(x,1)-v(x-1,1)-c]^+\\
&\quad +[v(x-1,1)-v(x-2,1)-c]^+\\
&\leqslant \begin{cases} D^2v(x,1), & \text{if } v(x-1,1)-v(x-2,1)\leqslant c\\ D^2v(x,1)-D^2v(x,1), & \text{otherwise}\end{cases}\\
&\leqslant 0,
\end{aligned}$$

where the first inequality results from $C1$ of $v(x,1)$. As $v(x,1)$ is a concave function of x, $D^2v(x+1,1)=Dv(x+1,1)-Dv(x,1)\leq 0$, and the second inequality results from $Dv(x-1,1)\geqslant Dv(x,1)$. So,

if $Dv(x-1,1)\leqslant c$, then $Dv(x,1)\leqslant c$ and

$$D^2v(x,1)-[Dv(x,1)-c]^+ + [Dv(x-1,1)-c]^+ = D^2v(x,1);$$

otherwise,

$$\begin{aligned}
&-[Dv(x,1)-c]^+ + [Dv(x-1,1)-c]^+\\
&\leqslant -[Dv(x,1)-c]+[Dv(x-1,1)-c]\\
&= -D^2v(x,1).
\end{aligned}$$

For H, we have

$$\begin{aligned}
D^2Hv(x,1) &= \lambda_1D^2v(x-1,1)+\lambda_2D^2v(x,1)\\
&\quad +(\lambda_2-\lambda_1)D^2[\xi_{12}-Dv(x,1)]^+\\
&= \lambda_1D^2v(x-1,1)\\
&\quad +\lambda_2D^2v(x,1)+(\lambda_2-\lambda_1)[\xi_{12}-Dv(x,1)]^+\\
&\quad -2(\lambda_2-\lambda_1)[\xi_{12}-Dv(x-1,1)]^+\\
&\quad +(\lambda_2-\lambda_1)[\xi_{12}-Dv(x-2,1)]^+\\
&\leqslant (\lambda_2-\lambda_1)D^2v(x,1)+(\lambda_2-\lambda_1)[\xi_{12}-Dv(x,1)]^+\\
&\quad -(\lambda_2-\lambda_1)[\xi_{12}-Dv(x-1,1)]^+,
\end{aligned}$$

and because of $C1$ of $v(x,1), D^2v(x,1)\leqslant 0, D^2v(x-1,1)\leqslant 0$,

$$\begin{aligned}
&Dv(x-1,1)\leqslant Dv(x-2,1),\\
&\xi_{12}-Dv(x-1,1)\geqslant \xi_{12}-Dv(x-2,1),\\
&[\{\xi_{12}-Dv(x-1,1)\}]^+\geqslant \{\xi_{12}-Dv(x-2,1)\}^+,
\end{aligned}$$

the inequality holds.

Then, if $Dv(x,1)>\xi_{12}$, as $Dv(x,1)\leqslant Dv(x-1,1)$, then $D^2Hv(x,1)\leqslant(\lambda_2-\lambda_1)D^2v(x,1)$;

otherwise,

$$\begin{aligned}
&(\lambda_2-\lambda_1)[\xi_{12}-Dv(x,1)]^+ - (\lambda_2-\lambda_1)[\xi_{12}-Dv(x-1,1)]^+\\
&\leqslant -(\lambda_2-\lambda_1)D^2v(x,1).
\end{aligned}$$

As a result, $D^2Hv(x,1)\leqslant 0$.

Similarly, we can show that $D^2Hv(x,1)\leqslant 0$.

As $Tv(x,i)$ is just the sum of concave functions that involve $-g(x)$, $v(x,i)$, $Av(x,1)$, and $Hv(x,i)$, it is also concave. So we have

$$DTv(x+1,i)-DTv(x,i)\leqslant 0,\ \text{for } i=0,1.$$

Verification of C2:

We prove $Tv(x,1)\geqslant Tv(x,0)$ first, for any x.

By application of $C2$, we obviously have $Av(x,1)\geqslant v(x,0)$ and $Hv(x,1)\geqslant Hv(x,0)$. So the first inequality of $C2$ holds.

In order to get $DTv(x,1)\leqslant DTv(x,0)$, we substitute the following specific expressions of $Tv(x,1)$ and $Tv(x,0)$ into the inequality

$$\begin{aligned}
Tv(x,1) &= -g(x)+q_0v(x,0)+q_1v(x,1)+Hv(x,1)+rAv(x,1),\\
Tv(x,0) &= -g(x)+q_0v(x,0)+q_1v(x,1)+rv(x,1)+Hv(x,0).
\end{aligned}$$

We divide this proof into two parts. In order to prove that $DTv(x,1)\leqslant DTv(x,0)$, we prove $DAv(x,1)\leqslant Dv(x,1)$ and $DHv(x,1)\leqslant DPv(x,0))$.

$$\begin{aligned}
&DAv(x,1)\leqslant Dv(x,1),\\
\iff\ & v(x,1)+[Dv(x+1,1)-c]^+ - v(x-1,1)\\
&\quad -[Dv(x,1)-c]^+\leqslant v(x,1)-v(x-1,1),\\
\iff\ & [Dv(x+1,1)-c]^+\leqslant [Dv(x,1)-c]^+.
\end{aligned}$$

Because of $C1$ of $v(x,1), Dv(x+1,1)\leqslant Dv(x,1)$, we have $[Dv(x+1,1)-c]^+\leqslant[Dv(x,1)-c]^+$.

$$\begin{aligned}
&DHv(x,1)\leqslant DHv(x,0)\\
\iff\ & DHv(x,1)-DHv(x,0)\leqslant 0\\
\iff\ & \lambda_1Dv(x-1,1)+\lambda_2Dv(x,1)-\lambda_1Dv(x-1,0)-\lambda_2Dv(x,0)\\
&\quad +(\lambda_2-\lambda_1)\{D[\xi_{12}-Dv(x,1)]^+ - D[\xi_{12}-Dv(x,0)]^+\}\leqslant 0,
\end{aligned}$$

$$\begin{aligned}
DHv(x,1)-DHv(x,0) &= \begin{cases} p(v)+(\lambda_2-\lambda_1)D[Dv(x,0)-Dv(x,1)],\\ \text{if } Dv(x,0)\leqslant\xi_{12}\\ p(v)+(\lambda_2-\lambda_1)[Dv(x,0)-Dv(x,1)],\\ \text{otherwise}\end{cases}\\
&= \begin{cases} \lambda_2(Dv(x-1,1)-Dv(x-1,0))\\ +\lambda_1(Dv(x,1)-Dv(x,0)),\\ \text{if } Dv(x,0)\leqslant\xi_{12}\\ \lambda_1(Dv(x-1,1)-Dv(x-1,0))\\ +\lambda_1(Dv(x,1)-Dv(x,0)),\\ \text{otherwise,}\end{cases}
\end{aligned}$$

where

$p(v)=\lambda_1Dv(x-1,1)+\lambda_2Dv(x,1)-\lambda_1Dv(x-1,0)-\lambda_2Dv(x,0)$.

From $C2, Dv(x,1)\leqslant Dv(x,0)$, and we get $DHv(x,1)\leqslant DHv(x,0)$, so $C2$ holds.

Verification of C3:

For $x<0$, by $C3$ that $Dv(x+1,1)>c$, we have the following observations.

$$\begin{aligned}
Dg(x+1) &= b>\gamma c,\\
DAv(x+1,1) &= Dv(x+1,1)+[Dv(x+2)-c]^+\\
&= Dv(x+1,1)+[Dv(x+2)-c]^+ - [Dv(x+1)-c]^+\\
&\geqslant Dv(x+1,1)-[Dv(x+1,1)-c]\\
&= c,
\end{aligned}$$

$$\begin{aligned} DHv(x+1,1) &= \lambda_1 Dv(x,1) + \lambda_2 Dv(x+1,1) + (\lambda_2 - \lambda_1) \\ &\quad \times \{[\xi_{12} - Dv(x+1,1)]^+ - [\xi_{12} - Dv(x,1)]^+\} \\ &\geqslant \lambda_1 c + \lambda_2 c. \end{aligned}$$

From $C2$, we already have $Dv(x,0) \geqslant Dv(x,1) \geqslant c$, then

$$\begin{aligned} DHv(x+1,1) &= \lambda_1 Dv(x,0) + \lambda_2 Dv(x+1,0) + (\lambda_2 - \lambda_1) \\ &\quad \times \{[\xi_{12} - Dv(x+1,0)]^+ - [\xi_{12} - Dv(x,0)]^+\} \\ &\geqslant \lambda_1 c + \lambda_2 c. \end{aligned}$$

Hence, we conclude that

$$DTv(x+1,1) > (\gamma + q_0 + q_1 + \lambda_1 + \lambda_2 + \mu)c = c, DTv(x+1,0) > c.$$

Verification of C4:

To evaluate $\lim_{x\to+\infty} DTv(x,i)$, by application of $C4$ and $g(x) = hx^+ - bx^-$, we obtain

$$\begin{aligned} \lim_{x\to+\infty} D[-g(x)] &= -h, \\ \lim_{x\to+\infty} DAv(x,1) &= \lim_{x\to+\infty} \{Dv(x,1) + D[Dv(x+1,1) - c]^+\} \\ &= \lim_{x\to+\infty} \{Dv(x,1) + [Dv(x+1,1) - c]^+ \\ &\quad - [Dv(x,1) - c]^+\} \\ &= \lim_{x\to+\infty} Dv(x,1) \\ &= -\frac{h}{\gamma}, \\ \lim_{x\to+\infty} DHv(x,i) &= \lim_{x\to+\infty} \{\lambda_1 Dv(x-1,i) + \lambda_2 Dv(x-1,i) \\ &\quad + (\lambda_2 - \lambda_1) D[\xi_{12} - Dvx, i]^+\} \\ &= \lim_{x\to+\infty} \{\lambda_1 Dv(x-1,i) + \lambda_2 Dv(x-1,i) \\ &\quad + (\lambda_2 - \lambda_1)[\xi_{12} - Dvx, i]^+ \\ &\quad - (\lambda_2 - \lambda_1)[\xi_{12} - Dvx - 1, i]^+\} \\ &= \lim_{x\to+\infty} [\lambda_1 Dv(x-1,i) + \lambda_2 Dv(x-1,i)] \\ &= -(\lambda_1 + \lambda_2)\frac{h}{\gamma}. \end{aligned}$$

Thus, we conclude that

$$\begin{aligned} \lim_{x\to+\infty} DTv(x,1) &= -h - (q_0 + q_1 + \lambda_1 + \lambda_2 + r)\frac{h}{\gamma} \\ &= -h - (1-\gamma)\frac{h}{\gamma} \\ &= -\frac{h}{\gamma}. \end{aligned}$$

Similarly,

$$\lim_{x\to+\infty} DTv(x,0) = -h - (q_0 + q_1 + \lambda_1 + \lambda_2 + r)\frac{h}{\gamma} = -\frac{h}{\gamma}.$$

We can similarly prove the other limits.

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